MAC 2311

**October 15, 2020** **Quiz 6**

**Remember to show <u>all</u> of your work.**

**Problem 1.** In all cases, differentiate and solve for $y'$:

**(a)** (3 points) $y = e^{3x}\arcsin(3x-2)$

**Sample Solution**: This is just simple differentiation.

$$y' = \frac{d}{dx}\big(e^{3x}\big) * \arcsin(3x-2) + e^{3x} * \frac{d}{dx}\big(\arcsin(3x-2)\big)$$

$$y' = 3e^{3x}\arcsin(3x-2) + e^{3x} * \frac{1}{\sqrt{1-(3x-2)^2}} * \frac{d}{dx}(3x-2)$$

$$y' = 3e^{3x}\arcsin(3x-2) + \frac{3e^{3x}}{\sqrt{1-(3x-2)^2}}$$

**(b)** (3 points) $-9 = 3x^2y^3 + x\tan(y)$

**Sample Solution**: This requires implicit differentiation.

$$0 = \frac{d}{dx}\big(3x^2\big) * y^3 + 3x^2 * \frac{d}{dx}\big(y^3\big) + \frac{d}{dx}\big(x\big) * \tan(y) + x * \frac{d}{dx}\big(\tan(y)\big)$$

$$0 = 6xy^3 + 3x^2 * 3y^2 * y' + \tan(y) + x\sec^2(y) * \frac{d}{dx}(y)$$

$$0 = 6xy^3 + 9x^2y^2y' + \tan(y) + xy'\sec^2(y)$$

$$-9x^2y^2y' - xy'\sec^2(y) = 6xy^3 + \tan(y)$$

$$y'(-9x^2y^2 - x\sec^2(y)) = 6xy^3 + \tan(y)$$

$$y' = \frac{6xy^3 + \tan(y)}{-9x^2y^2 - x\sec^2(y)}$$

**(c)** (4 points) $y = x^{2\cos(3x)}$

**Sample Solution**: This requires logarithmic differentiation.

$$y = x^{2\cos(3x)}$$

$$\ln(y) = \ln(x^{2\cos(3x)})$$

$$\ln(y) = 2\cos(3x)\ln(x)$$

$$\frac{d}{dx}\ln(y) = \frac{d}{dx}\big(2\cos(3x)\big) * \ln(x) + 2\cos(3x) * \frac{d}{dx}\big(\ln(x)\big)$$

$$\frac{1}{y} * y' = -2\sin(3x) * \frac{d}{dx}(3x) * \ln(x) + 2\cos(3x) * \frac{1}{x}$$

$$\frac{1}{y} * y' = -2\sin(3x) * 3 * \ln(x) + 2\cos(3x) * \frac{1}{x}$$

$$\frac{1}{y} * y' = -6\sin(3x)\ln(x) + \frac{2\cos(3x)}{x}$$

$$y' = y\left( -6\sin(3x)\ln(x) + \frac{2\cos(3x)}{x}\right)$$

$$y' = x^{2\cos(3x)}\left( -6\sin(3x)\ln(x) + \frac{2\cos(3x)}{x}\right)$$