MAC 2311

October 29, 2020 Quiz 8

Remember to show all of your work.

Problem 1. Determine all critical points of $f(x) = 4\sin^2(x+1) + \cos^2(x+1)$ on the interval $[0, 2\pi]$.

NOTE: this question was thrown out due to a typo and also because Dr. York mentioned it would not be on the quiz...and yet here it was. Woops! None the less, the solution is as follows:

Sample Solution: this comes from finding where $f'(x) = 0$ or where $f'(x)$ is undefined. We have

$$\begin{aligned} f'(x) &= 8\sin(x+1)\cos(x+1) - 2\sin(x+1)\cos(x+1) \\ &= 6\sin(x+1)\cos(x+1) \end{aligned}$$

Note that $f'(x)$ is defined everywhere, so we only need to find where $f'(x) = 0$. Now, $\sin(x) = 0$ when $x = n\pi$ for any integer n. Likewise, $\cos(x) = 0$ when $x = \frac{(2n+1)\pi}{2} = n + \frac{\pi}{2}$ for any integer n. Because we only need $x \in [0, 2\pi]$, this gives

$$x + 1 = \tfrac{\pi}{2}, \pi, \tfrac{3\pi}{2}, 2\pi$$

and thus the critical points are $x = \frac{\pi}{2} - 1, \pi - 1, \frac{3\pi}{2} - 1, 2\pi - 1$

Problem 2. Compute dy and Δy of the function $y = e^{x^2}$ as x goes from 1 to 0.

Sample Solution We have $\Delta y = f(b) - f(a) = e^0 - e^1 = 1 - e$

Also, $dy = f'(a)dx = f'(a)(b-a)$

Here, $f'(x) = 2xe^{x^2}$, and $a = 1$ so $f'(a) = f'(1) = 2(1)e^{1^2} = 2e$.

Thus, $dy = f'(a)(b-a) = 2e(0-1) = -2e$.