MAC 2311

January 19, 2020 **Quiz 1 - Keeran**

Remember to show all of your work.

Problem 1. Determine the following limits, or write DNE:

(a) (2 points) $\lim_{x\to 5} \dfrac{1}{x-5}$

Sample Solution: Here we have to check the left- and right-hand limits separately.
Consider $\lim_{x\to 5^-} \dfrac{1}{x-5}$. We'll take $x = 4.5, 4.9$, and 4.99.

Let $x = 4.5$, then $\dfrac{1}{x-5} = \dfrac{1}{4.5-5} = \dfrac{1}{-0.5} = -2$.

Let $x = 4.9$, then $\dfrac{1}{x-5} = \dfrac{1}{4.9-5} = \dfrac{1}{-0.1} = -10$.

Let $x = 4.99$, then $\dfrac{1}{x-5} = \dfrac{1}{4.99-5} = \dfrac{1}{-0.01} = -100$.

Repeating this process gives $\lim_{x\to 5^-} \dfrac{1}{x-5} = -\infty$

Now consider $\lim_{x\to 5^+} \dfrac{1}{x-5}$. We'll take $x = 5.5, 5.1$, and 5.01.

Let $x = 5.5$, then $\dfrac{1}{x-5} = \dfrac{1}{5.5-5} = \dfrac{1}{0.5} = 2$.

Let $x = 5.1$, then $\dfrac{1}{x-5} = \dfrac{1}{5.1-5} = \dfrac{1}{0.1} = 10$.

Let $x = 5.01$, then $\dfrac{1}{x-5} = \dfrac{1}{5.01-5} = \dfrac{1}{0.01} = 100$.

Repeating this process gives $\lim_{x\to 5^+} \dfrac{1}{x-5} = \infty$

Thus,

$$\lim_{x\to 5^-} \frac{1}{x-5} = -\infty \neq \infty = \lim_{x\to 5^+} \frac{1}{x-5}$$

and because the left- and right-hand limits are different, $\lim_{x\to 5} \dfrac{1}{x-5}$ does not exist.

(b) (2 points) $\lim_{x\to 1} \quad \frac{4}{(x-1)^2}$

Sample Solution: Similarly, here we also have to check the left- and right-hand limits separately.

Consider $\lim_{x\to 1^-} \frac{4}{(x-1)^2}$. We'll take $x = 0.5, 0.9$, and 0.99.

Let $x = 0.5$, then $\frac{4}{(x-1)^2} = \frac{4}{(0.5-1)^2} = \frac{4}{(-0.5)^2} = \frac{4}{0.25} = 16.$

Let $x = 0.9$, then $\frac{4}{(x-1)^2} = \frac{4}{(0.9-1)^2} = \frac{4}{(-0.1)^2} = \frac{4}{0.01} = 400.$

Let $x = 0.99$, then $\frac{4}{(x-1)^2} = \frac{4}{(0.99-1)^2} = \frac{4}{(-0.01)^2} = \frac{4}{0.0001} = 40000.$

Repeating this process gives $\lim_{x\to 1^-} \frac{4}{(x-1)^2} = \infty$

Now consider $\lim_{x\to 1^+} \frac{4}{(x-1)^2}$. We'll take $x = 1.5, 1.1$, and 1.01.

Let $x = 1.5$, then $\frac{4}{(x-1)^2} = \frac{4}{(1.5-1)^2} = \frac{4}{(0.5)^2} = \frac{4}{0.25} = 16.$

Let $x = 1.1$, then $\frac{4}{(x-1)^2} = \frac{4}{(1.1-1)^2} = \frac{4}{(0.1)^2} = \frac{4}{0.01} = 400.$

Let $x = 1.01$, then $\frac{4}{(x-1)^2} = \frac{4}{(1.01-1)^2} = \frac{4}{(0.01)^2} = \frac{4}{0.0001} = 40000.$

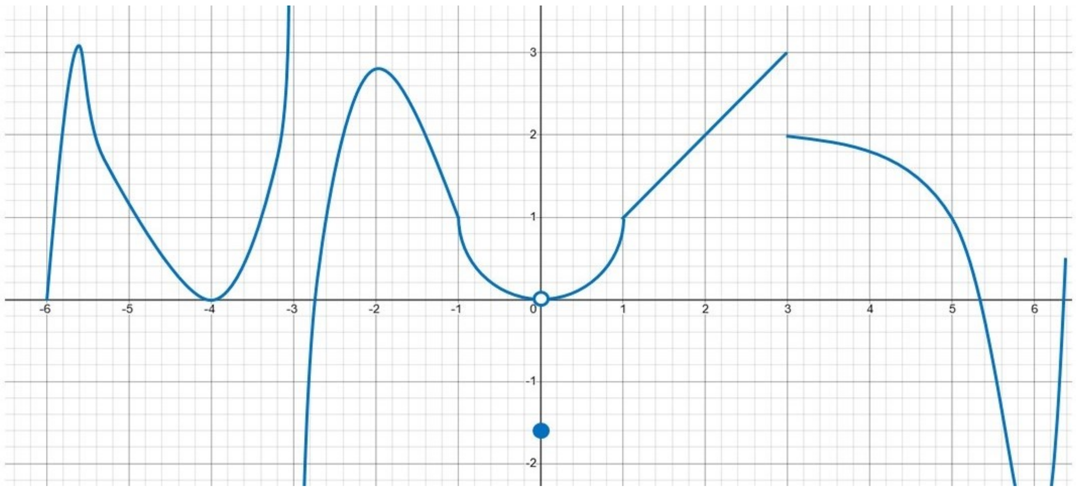
Repeating this process gives $\lim_{x\to 1^+} \frac{4}{(x-1)^2} = \infty$

Thus,

$$\lim_{x\to 1^-} \frac{4}{(x-1)^2} = \infty = \lim_{x\to 1^+} \frac{4}{(x-1)^2}$$

and because the left- and right-hand limits are the same, $\lim_{x\to 1} \frac{4}{(x-1)^2} = \infty$.

Problem 2. Use the graph below to determine the following limits, or write DNE. Assume lines off the graph go to ∞ or $-\infty$. (1 point each):



(a) $\lim_{x \to -4}$ **(b)** $\lim_{x \to -3}$ **(c)** $\lim_{x \to 0}$ **(d)** $\lim_{x \to 1}$ **(e)** $\lim_{x \to 3}$ **(f)** $\lim_{x \to 6}$

Sample Solutions

(a) $\lim_{x \to -4} = 0$

(b) $\lim_{x \to -3} =$ DNE, because $\lim_{x \to -3^-} = \infty$ and $\lim_{x \to -3^+} = -\infty$

(c) $\lim_{x \to 0} = 0$

(d) $\lim_{x \to 1} = 1$

(e) $\lim_{x \to 3} =$ DNE, because $\lim_{x \to 3^-} = 3$ and $\lim_{x \to 3^+} = 2$

(f) $\lim_{x \to 6} = -\infty$