MAC 2311

January 19, 2020 **Quiz 1 - York**

Remember to show <u>all</u> of your work.

Problem 1. (5 points) Determine the following limits, or write DNE:

(a) $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x - 2}$

This problem is an application of factoring and canceling:

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+1)}{x-2} = \lim_{x \to 2} \; x + 1 = 2 + 1 = 3$$

(b) $\lim_{x \to 1} \dfrac{4}{(x-1)^2}$

Sample Solution: Here we have to check the left- and right-hand limits separately.

Consider $\lim_{x \to 1^-} \dfrac{4}{(x-1)^2}$. We'll take $x = 0.5, 0.9,$ and 0.99.

Let $x = 0.5$, then $\dfrac{4}{(x-1)^2} = \dfrac{4}{(0.5-1)^2} = \dfrac{4}{(-0.5)^2} = \dfrac{4}{0.25} = 16.$

Let $x = 0.9$, then $\dfrac{4}{(x-1)^2} = \dfrac{4}{(0.9-1)^2} = \dfrac{4}{(-0.1)^2} = \dfrac{4}{0.01} = 400.$

Let $x = 0.99$, then $\dfrac{4}{(x-1)^2} = \dfrac{4}{(0.99-1)^2} = \dfrac{4}{(-0.01)^2} = \dfrac{4}{0.0001} = 40000.$

Repeating this process gives $\lim_{x \to 1^-} \dfrac{4}{(x-1)^2} = \infty$

Now consider $\lim_{x \to 1^+} \dfrac{4}{(x-1)^2}$. We'll take $x = 1.5, 1.1,$ and 1.01.

Let $x = 1.5$, then $\dfrac{4}{(x-1)^2} = \dfrac{4}{(1.5-1)^2} = \dfrac{4}{(0.5)^2} = \dfrac{4}{0.25} = 16.$

Let $x = 1.1$, then $\dfrac{4}{(x-1)^2} = \dfrac{4}{(1.1-1)^2} = \dfrac{4}{(0.1)^2} = \dfrac{4}{0.01} = 400.$

Let $x = 1.01$, then $\dfrac{4}{(x-1)^2} = \dfrac{4}{(1.01-1)^2} = \dfrac{4}{(0.01)^2} = \dfrac{4}{0.0001} = 40000.$

Repeating this process gives $\lim_{x \to 1^+} \dfrac{4}{(x-1)^2} = \infty$

Thus,

$$\lim_{x \to 1^-} \frac{4}{(x-1)^2} = \infty = \lim_{x \to 1^+} \frac{4}{(x-1)^2}$$

and because the left- and right-hand limits are the same, $\lim_{x \to 1} \dfrac{4}{(x-1)^2} = \infty.$

Problem 2. (5 points)

Let $\lim_{x\to a} f(x) = 1$, $\lim_{x\to a} g(x) = 2$, $\lim_{x\to a} h(x) = 3$, and $\lim_{x\to a} j(x) = 4$. Calculate the following:

$$\lim_{x\to a}\left[\frac{f(x) - 2g(x) + (h(x))^2}{g(x) + j(x)}\right]$$

Sample Solution: Here we'll utilize limit laws. We have:

$$\begin{aligned}
\lim_{x\to a}\left[\frac{f(x) - 2g(x) + (h(x))^2}{g(x) + j(x)}\right] &= \frac{\lim_{x\to a}\big[f(x)\big] - \lim_{x\to a}\big[2g(x)\big] + \lim_{x\to a}\big[(h(x))^2\big]}{\lim_{x\to a}\big[g(x)\big] + \lim_{x\to a}\big[j(x)\big]} \\
&= \frac{\lim_{x\to a}\big[f(x)\big] - 2\Big[\lim_{x\to a}\big[g(x)\big]\Big] + \Big[\lim_{x\to a}\big[h(x)\big]\Big]^2}{\lim_{x\to a}\big[g(x)\big] + \lim_{x\to a}\big[j(x)\big]} \\
&= \frac{1 - 2(2) + (3)^2}{2 + 4} \\
&= \frac{1 - 4 + 9}{6} \\
&= \frac{6}{6} \\
&= 1
\end{aligned}$$