MAC 2311

January 26, 2021 **Quiz 2 - Keeran & York**

Remember to show <u>all</u> of your work.

Problem 1. Find $\lim_{x\to 0} 2x^2 \sin\left(\frac{4}{x^2}\right)$

Sample Solution: This problem is all about using the Squeeze Theorem. Consider that $\sin$ is a bounded function, i.e. $-1 \leq \sin(\alpha) \leq 1$ for any function or number α. Therefore, we have

$$-1 \leq \sin\left(\frac{4}{x^2}\right) \leq 1$$
$$-2x^2 \leq 2x^2 \sin\left(\frac{4}{x^2}\right) \leq 2x^2$$
$$\lim_{x\to 0} -2x^2 \leq \lim_{x\to 0} 2x^2 \sin\left(\frac{4}{x^2}\right) \leq \lim_{x\to 0} 2x^2$$

Now, direct substitution on the left and right sides of the inequality yields

$$0 \leq \lim_{x\to 0} 2x^2 \sin\left(\frac{4}{x^2}\right) \leq 0$$

and thus $\lim_{x\to 0} 2x^2 \sin\left(\frac{4}{x^2}\right) = 0$ by the Squeeze Theorem.

Problem 2. Evaluate $\lim_{x\to 1} \frac{-x^2+2x-1}{|x-1|}$.

Sample Solution Here because we're dealing with an absolute value, it's necessary to construct a piecewise function. Consider

$$x - 1 = 0 \to x = 1$$

so the split in our piecewise function will be at $x = 1$. Thus, we have

$$\lim_{x\to 1} \frac{-x^2+2x-1}{|x-1|} = \begin{cases} \lim_{x\to 1} \frac{-x^2+2x-1}{x-1} & \text{for } x \geq 1 \\ \\ \lim_{x\to 1} \frac{-x^2+2x-1}{-(x-1)} & \text{for } x < 1 \end{cases}$$

Now we have to consider the left- and right-side limits.

Taking the limit from the right, we use the top part of our piecewise function (i.e. for $x \geq 1$):

$$\begin{aligned}
\lim_{x\to 1^+} \frac{-x^2+2x-1}{|x-1|} &= \lim_{x\to 1^+} \frac{-x^2+2x-1}{x-1} \\
&= \lim_{x\to 1^+} \frac{(-x+1)(x-1)}{x-1} \\
&= \lim_{x\to 1^+} -x+1 \\
&= -(1)+1 \\
&= 0
\end{aligned}$$

Taking the limit from the left, we use the bottom part of our piecewise function (i.e. for $x < 1$):

$$\begin{aligned}
\lim_{x\to 1^-} \frac{-x^2+2x-1}{|x-1|} &= \lim_{x\to 1^-} \frac{-x^2+2x-1}{-(x-1)} \\
&= \lim_{x\to 1^-} \frac{(-x+1)(x-1)}{-x+1} \\
&= \lim_{x\to 1^-} x-1 \\
&= (1)-1 \\
&= 0
\end{aligned}$$

Thus, because the left- and right-side limits equal,

$$\lim_{x\to 1} \frac{-x^2+2x-1}{|x-1|} = 0$$