MAC 2311

February 2, 2021 **Quiz 3 - Keeran & York**

Remember to show <u>all</u> of your work.

Problem 1. Find $\lim_{x\to\infty} \frac{3x^2}{(2x-1)^2}$

Sample Solution: Consider

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3x^2}{(2x-1)^2} = \lim_{x\to\infty} \frac{3x^2}{4x^2-4x+1}$$

Now, trying direct substitution yields $\frac{\infty}{\infty}$, so we have to simplify a bit more. Here, we'll utilize the trick of dividing by the highest power of x in the problem, which here is x^2. We have

$$\lim_{x\to\infty} \frac{3x^2}{4x^2-4x+1} = \lim_{x\to\infty} \frac{\frac{3x^2}{x^2}}{\frac{4x^2-4x+1}{x^2}} = \lim_{x\to\infty} \frac{\frac{3x^2}{x^2}}{\frac{4x^2}{x^2} - \frac{4x}{x^2} + \frac{1}{x^2}} = \lim_{x\to\infty} \frac{3}{4 - \frac{4}{x} + \frac{1}{x^2}}$$

Note that as $x \to \infty$, $\frac{4}{x} \to 0$ and $\frac{1}{x^2} \to 0$, and thus

$$\lim_{x\to\infty} \frac{3}{4 - \frac{4}{x} + \frac{1}{x^2}} = \frac{3}{4-0-0} = \frac{3}{4}$$

Problem 2. Given $s(t) = t^2 + 5$, find

- The average velocity from $t = 3$ to $t = 4$
- The instantaneous velocity at $t = 3$.

Sample Solution: First, we utilize the formula for average velocity: $\frac{s(b)-s(a)}{b-a}$. Based on the problem, we have

- $a = 3$
- $b = 4$
- $s(a) = s(3) = 3^2 + 5 = 14$
- $s(b) = s(4) = 4^2 + 5 = 21$

And thus, plugging in to the formula gives

$$\frac{s(b)-s(a)}{b-a} = \frac{21-14}{4-3} = \frac{7}{1} = 7$$

So the average velocity from $t = 3$ to $t = 4$ is 7.

Option 1: Now, to find instantaneous velocity we utilize the formula $\lim_{h \to 0} \frac{s(t+h)-s(t)}{h}$ (you may have seen this formula before with Δx and f; this is the same formula, just using the variables given in the problem.) We have:

$$\begin{aligned}
\lim_{h \to 0} \frac{s(t+h)-s(t)}{h} &= \lim_{h \to 0} \frac{\left[(t+h)^2+5\right]-\left[t^2+5\right]}{h} \\
&= \lim_{h \to 0} \frac{\left[t^2+2th+h^2+5\right]-\left[t^2+5\right]}{h} \\
&= \lim_{h \to 0} \frac{t^2+2th+h^2+5-t^2-5}{h} \\
&= \lim_{h \to 0} \frac{2th+h^2}{h} \\
&= \lim_{h \to 0} 2t+h \\
&= 2t
\end{aligned}$$

But now, remember that the problem is asking about time $t = 3$. Therefore, the instantaneous velocity at $t = 3$ is 6.

Option 2: Now, to find instantaneous velocity we utilize the formula $\lim_{h \to 0} \frac{s(t+h)-s(t)}{h}$ at time $t = 3$ (you may have seen this formula before with Δx and f; this is the same formula, just using the variables given in the problem.) We have:

$$\begin{aligned}
\lim_{h \to 0} \frac{s(3+h)-s(3)}{h} &= \lim_{h \to 0} \frac{\left[(3+h)^2+5\right]-\left[3^2+5\right]}{h} \\
&= \lim_{h \to 0} \frac{\left[3^2+6h+h^2+5\right]-\left[3^2+5\right]}{h} \\
&= \lim_{h \to 0} \frac{9+6h+h^2+5-9-5}{h} \\
&= \lim_{h \to 0} \frac{6h+h^2}{h} \\
&= \lim_{h \to 0} 6+h \\
&= 6
\end{aligned}$$

Therefore, the instantaneous velocity at $t = 3$ is 6.