MAC 2311

March 2, 2021 **Quiz 6 - Keeran & York**

Remember to show <u>all</u> of your work.

Problem 1. Find the derivative of $y = \tan(x)^{e^x}$ (Solve for y', but no need to simplify).

Sample Solution: Here, I definitely recommend using logarithmic differentiation. Therefore, we start by taking the $\ln$ of both sides:

$$\ln(y) = \ln\left(\tan(x)^{e^x}\right)$$

and now we can utilize logarithm rules to move the exponent out front:

$$\ln(y) = e^x \ln\left(\tan(x)\right)$$

And now we can derive, utilizing a combination of product and chain rule:

$$\begin{aligned}
\frac{1}{y} * y' &= \frac{d}{dx} e^x * \ln(\tan(x)) + e^x * \frac{d}{dx} \ln(\tan(x)) \\
&= e^x \ln(\tan(x)) + e^x \frac{1}{\tan(x)} * \sec^2(x) \\
&= e^x \ln(\tan(x)) + \frac{e^x \sec^2(x)}{\tan(x)}
\end{aligned}$$

Note that $\frac{d}{dx} \ln(\tan(x))$ requires chain rule to solve. Now, we have to solve for y':

$$\begin{aligned}
\frac{1}{y} * y' &= e^x \ln(\tan(x)) + \frac{e^x \sec^2(x)}{\tan(x)} \\
y' &= \left(e^x \ln(\tan(x)) + \frac{e^x \sec^2(x)}{\tan(x)}\right) y \\
&= \left(e^x \ln(\tan(x)) + \frac{e^x \sec^2(x)}{\tan(x)}\right) \tan(x)^{e^x}
\end{aligned}$$

where in the last line we substitute the original expression in for y.

Problem 2. A particle is moving on a line with position given by

$$s(t) = \tfrac{1}{3}t^3 - 2t^2 - 5t + 1$$

Find the values of t where

- the particle is standing still
- the particle is moving forward
- the particle is moving backward

This problem is all about determining values having to do with the velocity. We start by deriving to find the velocity:

$$\begin{aligned} s(t) &= \frac{1}{3}t^3 - 2t^2 - 5t + 1 \\ \longrightarrow v(t) &= t^2 - 4t - 5 \\ &= (t+1)(t-5) \end{aligned}$$

Therefore, we can answer (a) from here: the velocity is zero at $t = -1$ and $t = 5$, but remember that we don't consider negative time. Therefore, the particle is still at $t = 5$.

Now, to find where the particle is moving backwards, we have to pick numbers on either side of $t = 5$ and see if the values of the velocity are positive or negative. Consider:

$$t = 1 \longrightarrow v(t) = (1)^2 - 4(1) - 5 = -8 < 0, \text{ so backwards}$$
$$t = 6 \longrightarrow v(t) = (6)^2 - 4(6) - 5 = 7 > 0, \text{ so forwards}$$

Therefore, for any t values less than 5, the particle moves backwards (i.e. $0 \le t < 5$, i.e. $[0, 5)$) and for any t values greater than 5, the particle moves forwards (i.e. $t > 5$, i.e. $(5, \infty)$).