

MAC1105 Week 11 Discussion

Module 11: Modeling

(1) A growth rate of the population of wolves in an area of Yellowstone National Park can be modeled using the following equation:

$$P(t) = \frac{3213}{3 + 14e^{-0.2t}}$$

where P is the population and t is the time in years.

- (a) Find the initial population of wolves.

Answer: $P(0) = 189$ wolves

- (b) Find the estimated population of wolves after 10 years have passed.

Answer: $P(10) = 656$ wolves

- (c) How many years need to pass for the population to surpass 1000 wolves?

Answer: 20.93 years

- (2) The half-life of fluorine-18 is 109.771 minutes.

- (a) Find an equation for the amount of fluorine-18 remaining as a function of time, t .

Answer: $A = A_0 e^{-0.00631t}$

- (b) What percentage of fluorine-18 remains after 6 minutes?

Answer: 96.3%

- (c) How long do we need to wait for only 1% of the original amount of fluorine-18 to remain?

Answer: 729.8 minutes

(3) While no one has ever been able to obtain a temperature of absolute zero, let's suppose that it is possible. Suppose a banana bread was just baked in an oven set to 200°C, and immediately placed in a freezer at -273°C. After 5 minutes, the bread has cooled to 75°C.

- (a) How long does the bread need to remain in the freezer before it is frozen?

Answer: 8.95 minutes

- (b) After how many minutes will the temperature of the bread be -37°C?

Answer: 11.32 minutes

- (c) How long will it take the bread to reach -272°C? -272.9°C? -272.99°C? -272.999°C?

Answer: 100.31 minutes, 137.81 minutes, 175.31 minutes, 212.81 minutes

- (d) Will the bread ever actually reach a temperature of -273°C? Why or why not?

Answer: No. Consider the equation if the bread was -273°C: $-273 = 473e^{-0.0614t} - 273$. This equation then becomes $0 = e^{-0.0614t}$, and thus in order to solve for t we need $\ln(0)$, which does not exist. To see it another way, think about the graph of the natural log function. As x approaches 0 from the right, y approaches $-\infty$. Therefore $x = 0$ is a vertical asymptote, and therefore the graph never actually touches zero. Thus, the temperature of the bread can never actually equal the temperature of the freezer.