MAC 2311

March 30, 2021 | Quiz 10 - Keeran & York

Remember to show <u>all</u> of your work.

Problem 1. Let $f(x) = \dfrac{x^3 - 16}{x}$. Then $f'(x) = \dfrac{2x^3 + 16}{x^2}$ and $f''(x) = \dfrac{2x^3 - 32}{x^3}$.

(a) Determine the domain of $f(x)$.

Sample Solution: Just glancing at the function, it's clear that $f(x)$ is defined everywhere except at $x = 0$. Thus, the domain is $(-\infty, 0) \cup (0, \infty)$.

(b) Find any vertical / horizontal asymptotes of $f(x)$, if they exist.

Sample Solution:
Vertical: setting the denominator equal to zero, we see there is a vertical asymptote at $x = 0$.
Horizontal: These are found by taking the limit as $x \to \infty$, $x \to -\infty$. We have:

$$\lim_{x\to\infty} \frac{x^3 - 1}{x} \overset{LH}{=} \lim_{x\to\infty} 3x^2 = \infty$$

$$\lim_{x\to-\infty} \frac{x^3 - 1}{x} \overset{LH}{=} \lim_{x\to-\infty} 3x^2 = \infty$$

so there are no horizontal asymptotes.

(c) Identify any possible critical points.

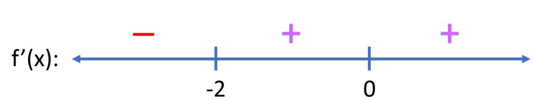
Sample Solution: Possible critical points are found by setting $f'(x) = 0$ or undefined. Because $x = 0$ isn't in our domain, we only need $f'(x) = 0$:

$$0 = \tfrac{2x^3+16}{x^2} \longrightarrow 2x^3 + 16 = 0 \longrightarrow x^3 = -8 \longrightarrow x = -2$$

and $f(-2) = \frac{(-2)^3-16}{-2} = \frac{-24}{-2} = 12$, so the possible critical point is $(-2, 12)$.

(d) Find the intervals on which $f(x)$ is increasing / decreasing.

Sample Solution: Increasing on $(-\infty, 2)$, decreasing on $(-2, 0) \cup (0, \infty)$.



(e) Identify any possible inflection points.

Sample Solution: Possible inflection points are found by setting $f''(x) = 0$ or undefined. Because $x = 0$ isn't in our domain, we only need $f''(x) = 0$:

$$0 = \tfrac{2x^3-32}{x^3} \longrightarrow 2x^3 - 32 = 0 \longrightarrow x^3 = 16 \longrightarrow x = \sqrt[3]{16}$$

and $f(\sqrt[3]{16}) = \frac{(\sqrt[3]{16})^3-16}{\sqrt[3]{16}} = \frac{0}{\sqrt[3]{16}} = 0$, so the possible critical point is $(\sqrt[3]{16}, 0)$.

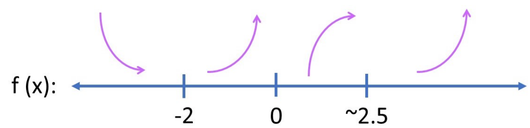
(f) Find the intervals on which $f(x)$ is concave up / concave down.

Sample Solution: Concave up on $(-\infty, 0) \cup (\sqrt[3]{16}, \infty)$, concave down on $(0, \sqrt[3]{16})$.

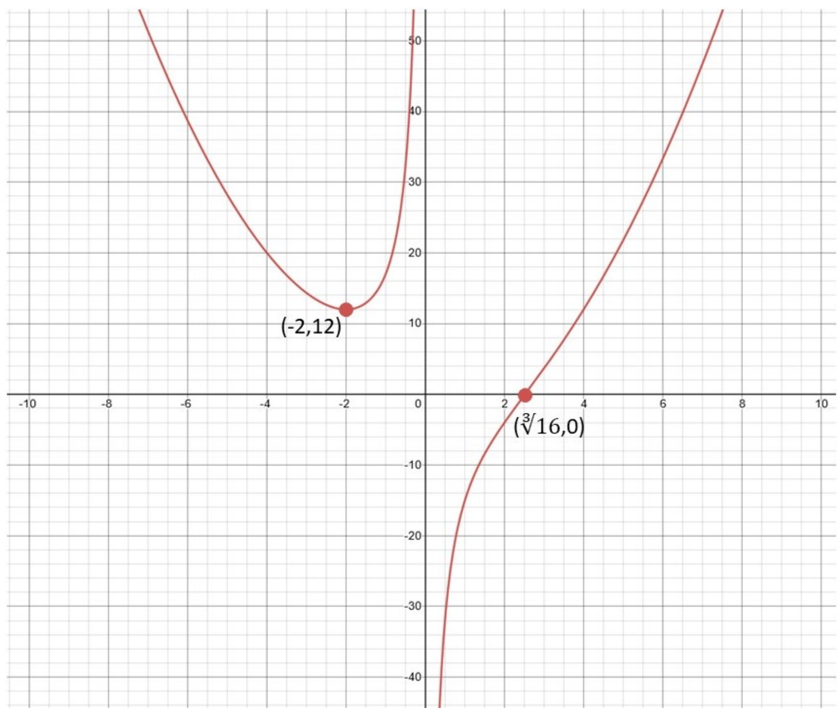
(g) Graph $f(x)$.

Hint for graphing: $\sqrt[3]{16} \approx 2.5$

Sample Solution: From our number lines of $f'(x)$ and $f''(x)$, we have



with important points $(-2, 12)$ and $(\sim 2.5, 0)$, and a vertical asymptote at $x = 0$. Thus,



Problem 2. (1 point) In honor of April Fool's Day on Thursday, what's your favorite joke / pun?
Sample Solution One of my favorite jokes:

What did the shy pebble wish for? That she could be a little boulder!