Parametric Curve And Polar Co-ordinates

DEC 1

Slope $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ @ $\frac{dx}{dt} \neq 0$

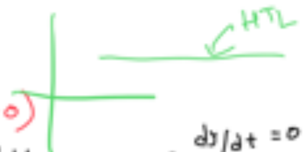
Concavity $\frac{d^2y}{dx^2} = \frac{d(dy/dx)}{dx}$

$$= \frac{d(dy/dx)/dt}{dx/dt}$$

Arc Length, $S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x(t) = t - x_0$

$y(t) = mt - y_0$

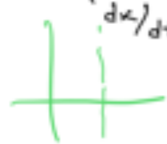


HTL (means slope $= 0$)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0 < \begin{matrix} dy/dt = 0 \\ dx/dt \neq 0 \end{matrix}$$

VTL (means slope $= \infty$)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \infty < \begin{matrix} dy/dt \neq 0 \\ dx/dt = 0 \end{matrix}$$

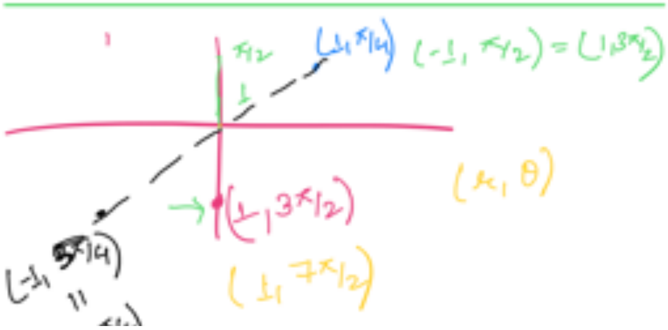


$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$, $x \neq 0$



$(1, \ldots)$

1) $x = 6$, $x(t) = ?$ $y(t) = ?$

$$\boxed{x(t) = 6, \quad y(t) = t}$$

$$m = \frac{dy/dt}{dx/dt} = \infty \quad \frac{\neq 0}{= 0} = \frac{1}{0} = \infty$$

2) $x(t) = e^t - t$
$y(t) = 4e^{t/2}$ Arc length = ?
$0 \le t \le 2$

$$L = \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$\cancel{4}2 e^{t/2} \cdot \frac{1}{\cancel{2}}$

$$= \int_0^2 \sqrt{(e^t - 1)^2 + (2e^{t/2})^2}\, dt$$

$$= \int_0^2 \sqrt{e^{2t} - 2e^t + 1 + 4e^t}\, dt$$

$$= \int_0^2 \sqrt{e^{2t} + 2e^t + 1}\, dt$$

$$= \int_0^2 \cancel{\sqrt{}}\ (e^t + 1)^{\cancel{2}}\, dt$$

$$= \int_0^2 (e^t + 1)\, dt$$

$$= e^t + t \Big|_0^2$$

$$= e^2 + 2 - (e^0 + 0)$$

$$= e^2 + 2 - 1$$

$$\boxed{= e^2 + 1}$$

Eqⁿ of tangent line.
$x(t) = \sin t$
$y(t) = e^t$ @ $t = 0$

$x_0 = \sin(0) = 0$ $\boxed{(0, 1)}$

$x_0 = \sin(0) = 0$ $\quad \boxed{(0, 1)}$

$y_0 = e^0 = 1$

$m = \frac{dy}{dx} = \frac{e^t}{\cos t}$ @ $t=0$ $= \frac{e^0}{\cos 0} = \frac{1}{1} = 1$

$y - y_0 = m(x - x_0)$

$y - 1 = 1(x - 0)$

$y - 1 = x$

$y = x + 1$

Options:

$x(t) = t - 1$
$y(t) = t$

OR

$x(t) = t$
$y(t) = t + 1$

When $t = 0$

$x(0) = -1$
$y(0) = 0$

$$\boxed{\begin{aligned} x(0) &= 0 \\ y(0) &= 0 + 1 = 1 \end{aligned}}$$

Convert $r = \tan\theta \sec\theta$ to cartesian coordinate.

$r = \frac{y}{x} \cdot \frac{1}{\cos\theta}$

$\Rightarrow x \, r\cos\theta = y$ $\qquad x = r\cos\theta$, $y = r\sin\theta$

$\Rightarrow x \cdot x = y$

$\therefore \boxed{y = x^2}$

$r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$\underbrace{r\cos\theta}_{x} \cos\theta = \sin\theta$

$\underbrace{r\cos\theta}_{x} = \underbrace{r\sin\theta}_{y}$

$\boxed{x^2 = y}$

$(x^2 + y^2)^2 = 2xy$ (Convert to Polar)

$\Rightarrow (r^2)^2 = 2 r\cos\theta \, r\sin\theta$

$\Rightarrow r^2 \cdot \cancel{r^2} = 2\cancel{r^2} \cos\theta \sin\theta$

$\Rightarrow \quad \boxed{r = \sin 2\theta}$

(3) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

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(8) $(x, y) = (-1, -\sqrt{3})$ (Convert to polar)

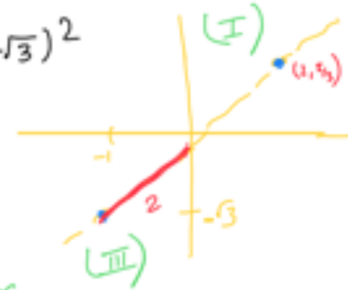
$r^2 = x^2 + y^2$

$= (-1)^2 + (-\sqrt{3})^2$

$= 1 + 3$

$r^2 = 4$

$\boxed{r = \pm 2}$



$\tan\theta = y/x$

$\tan\theta = \dfrac{\cancel{-}\sqrt{3}}{\cancel{-}1}$

$\tan\theta = \sqrt{3}$

$\boxed{\theta = \pi/3,\ 4\pi/3}$

When r is +ve,

$r = 2$

When r is −ve,

$r = -2$

$\boxed{(2, 4\pi/3) \qquad (-2, \pi/3)}$

(?) $x(t) = \sqrt{t}$

$y(t) = \dfrac{(t^2 - 4)}{4}, \quad t > 0$

Find the concavity @ $(2, 3)$

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\frac{1}{\cancel{4}}\cancel{2}t}{\frac{1}{\cancel{2}\sqrt{t}}}$

$= t\cdot\sqrt{t} = t\cdot t^{1/2}$

$= t^{3/2}$

$d\,(dy/dx) = d(t^{3/2})/dt$

$$\frac{}{dx} \qquad \frac{}{d(\sqrt{t})/dt}$$

$$= \frac{\frac{3}{\cancel{2}}\sqrt{t}}{\frac{1}{\cancel{2}\sqrt{t}}}$$

$$= 3\sqrt{t} \cdot \sqrt{t}$$

$$= 3t \qquad = 3 \cdot 4 \boxed{= 12}$$

$$y = \frac{t^2 - 4}{4} \qquad (2, 3)$$

$$3 = \frac{t^2 - 4}{4}$$

$$12 = t^2 - 4 \qquad \boxed{t = 4}$$

$$16 = t^2$$

$$t = \pm 4$$

$\boxed{\text{Concave Up}}$