DEC 8

CIRCLE FAMILY:

$r = a$, $r = 2a\sin\theta$, $r = 2a\cos\theta$

PETAL FAMILY

$r = a\sin(n\theta)$, $r = a\cos(n\theta)$

When n is even, there are $2n$ petals

When n is odd, there are n petals.

In $\sin\theta$, the first petal is in first quadrant

In $\cos\theta$, the first petal is in x-axis

CARDIOID FAMILY

$c = 1$ (heart), $c > 1$ (limacon)

$c < 1$ (dimple / dent)

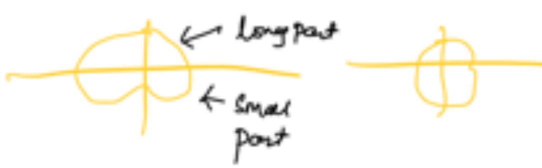
The long part of the picture is

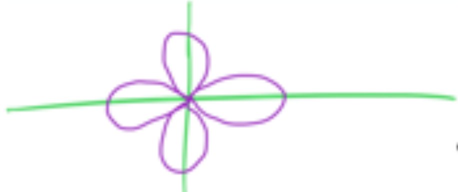
$r = a(1 + c\sin\theta)$ (on the top)

$r = a(1 - c\sin\theta)$ (on the bottom)

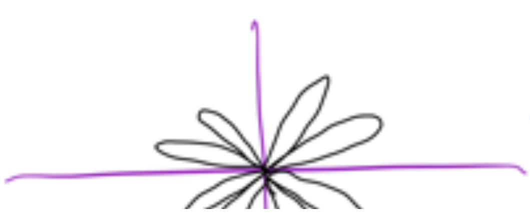
$r = a(1 + c\cos\theta)$ (on the right)

$r = a(1 - c\cos\theta)$ (on the left)





$r = \cos 2\theta$



$r = \sin 4\theta$

⊛ Area of a region that lies inside the 1st curve and outside the second curve

$r_1 = 2 + \sin\theta \qquad r_2 = 3\sin\theta$

$= 2\left(1 + \frac{1}{2}\sin\theta\right)$

θ	r_1
0	2
$\pi/2$	3
π	2
$3\pi/2$	1
2π	2

Right half:

$A_{\text{between}} = A_{\text{outside}} - A_{\text{inside}}.$

$$A_{\text{outside}} = \frac{1}{2}\int_{-\pi/2}^{\pi/2} (2+\sin\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left(4 + 4\sin\theta + \sin^2\theta\right) d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left(4 + 4\sin\theta + \frac{1}{2} - \frac{\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left(\frac{9}{2} + 4\sin\theta - \frac{\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\left[\frac{9}{2}\theta - 4\cos\theta - \frac{\sin 2\theta}{2\cdot 2}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}\left(\left(\frac{9}{2}\left(\frac{\pi}{2}\right) - 0 - 0\right) - \left(\frac{9}{2}\left(\frac{-\pi}{2}\right) - 0 - 0\right)\right)$$

$$= \frac{1}{2}\left[\frac{9\pi}{4} + \frac{9\pi}{4}\right]$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \frac{9\pi}{4} \qquad \boxed{I = \frac{9\pi}{4}}$$

$$A_{inside} = \frac{\pi r^2}{2} \qquad (r = 3/2)$$

$$= \frac{\pi}{2}\left(\frac{3}{2}\right)^2$$

$$= \frac{\pi \cdot 9}{8} \qquad \boxed{= \frac{9\pi}{8}}$$

$$A_{between} = 2 \cdot [A_{out} - A_{in}]$$

$$= 2 \cdot \left[\frac{9\pi}{4} - \frac{9\pi}{8}\right]$$

$$= 2\left[\frac{2 \cdot 9\pi - 9\pi}{8}\right]$$

$$= \cancel{2} \cdot \left[\frac{9\pi}{8_4}\right]$$

$$\boxed{= \frac{9\pi}{4}}$$

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(3) $\displaystyle\int_0^2 \ln(x^2+4)\,dx$

$\downarrow u$	$v \uparrow$
$\ln(x^2+4)$	$+1$
$= \frac{1}{x^2+4} \cdot 2x$	x

$$= x\ln(x^2+4) - \int \frac{2x}{x^2+4} \cdot x\,dx = \int \frac{2x^2}{x^2+4}dx$$

$$= 2\int \frac{x^2+4-4}{x^2+4}dx$$

$$\int \frac{1}{x^2+a^2}$$

$$= 2\int \frac{x^2+4}{x^2+4} - \frac{4}{x^2+4}\,dx$$

$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$

$= \frac{1}{a} \arctan\left(\frac{x}{a}\right)$

$$= 2\int 1 - 4\left(\frac{1}{x^2+4}\right) dx$$

$$= 2x - \cancel{2}4 \cdot \frac{1}{\cancel{2}} \arctan\left(\frac{x}{2}\right)$$

$$= x\ln(x^2+4) - 2x + 4\arctan\left(\frac{x}{2}\right) \Big|_0^2$$

$$= 2\ln(8) - 4 + \cancel{4} \cdot \frac{\pi}{\cancel{4}} - 0$$

$$\boxed{= 2\ln(8) - 4 + \pi}$$

$$= 2\ln(2^3) - 4 + \pi$$

$$\boxed{= 6\ln 2 - 4 + \pi}$$

If a_n is seq of positive number and $\sum_{n=1}^{\infty} a_n = 10$, which one is bigga?

ⓐ S_{2020}

ⓑ a_{2020}

ⓒ $\lim_{n\to\infty} a_n$

ⓓ $\lim_{N\to\infty} S_N$

$$\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_N = 10$$

big

Small $\lim_{n\to\infty} a_n < a_{2020} < S_{2020} < \lim_{N\to\infty} S_N$

$$\overbrace{a_1 + a_2 + a_3 + \cdots\cdots + a_n}^{S_{2020}} + a_{n+1} \cdots = 10$$

$\uparrow$ a_{2020}

If $a_n = \frac{1}{n^2}$

$a_{2020} = \frac{1}{(2020)^2} > 0$

Q. Find the R.O.C

$$\sum_{2}^{\infty} \frac{\ln n}{n^2} (2x-1)^n$$

$$= \lim_{n\to\infty} \left| \frac{\ln(n+1)\,\cancel{(2x-1)^n}\,(2x-1)}{\ln(n)\,\cancel{(2x-1)^n}} \cdot \frac{n^2}{(n+1)^2} \right|$$

$$= |2x-1| \lim_{n\to\infty} \left| \frac{\ln(n+1)}{\ln(n)} \cdot \frac{n^2}{(n+1)^2} \right|$$

$$= |2x-1| \underbrace{\lim_{n\to\infty} \left| \frac{\ln(n+1)}{\ln(n)} \right|}_{\text{L. Hospital}} \cdot \lim_{n\to\infty} \frac{n^2}{(n+1)^2}$$

$$= |2x-1| \lim_{n\to\infty} \left| \frac{1}{(n+1)} \cdot n \right| \cdot 1$$

$$= |2x-1| < 1$$

$$= 2\left|x - 1/2\right| < 1 \Rightarrow \left|x - \frac{1}{2}\right| < \frac{1}{2} = R$$