Nov 10

1) Power Series is a series of the form

$$\sum_{n=0}^{\infty} a_n (x-c)^n \quad , \quad \begin{array}{l} a_n = \text{seq.} \\ x = \text{variable} \\ c = \text{centre} \end{array}$$

2) Values for which x converge is called Interval of Convergence (I.O.C)

3) Half of length of I.O.C is called Radius of Convergence (R.O.C)

Geometric Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1 = R$$

I.O.C $-1 < x < 1$

$(-1, 1)$

$$\rightarrow \frac{1}{1-x} = 1 + x + x^2 + \cdots$$

Note: Ratio / Root , $L < 1$ Σ converge.

$L = 1$ (Inconclusive)

$$\circledast \ \frac{d}{dx}\left(\sum_{n=0} a_n (x-c)^n\right) = \sum_{n=0}^{\infty} a_n \frac{d}{dx}(x-c)^n$$

$$= \sum_{n=1}^{\infty} a_n \cdot n\,(x-c)^{n-1}$$

(coeff

⊛ R.O.C same.

→ I.O.C, (,) Don't check endpoints

[,] Check endpoints

$$\circledast \int \sum_{n=0}^{\infty} a_n (x-c)^n = \sum_{n=0}^{\infty} a_n \frac{(x-c)^{n+1}}{n+1} + C$$

⊛ R.O.C same

I.O.C → (,) Check endpoints

$\rightarrow [\ ,\ ]$ Don't check endpts

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⑧ $\dfrac{3}{x^2+x-2}$ Find power series representation, R.O.C, I.O.C ?

$\Rightarrow \dfrac{3}{(x-1)(x+2)}$

$\Rightarrow \dfrac{3}{(x-1)(x+2)}$, $\Rightarrow \dfrac{A}{x-1} + \dfrac{B}{x+2}$

Use cover up method.

$x=1$, $\dfrac{3}{1+2} = A \ \boxed{=1}$

$x=-2$, $\dfrac{3}{-2-1} = \boxed{B=-1}$

$\Rightarrow \dfrac{1}{x-1} - \dfrac{1}{x+2}$ (Goal: $\frac{1}{1-x}$)

ⓐ $\dfrac{1}{x-1} = \dfrac{-1}{(1-x)} = -\sum_{n=0}^{\infty} x^n$,

$|x| < 1 = R.O.C$

$I.O.C = (-1,\ 1)$

ⓑ $\dfrac{-1}{x+2} = \dfrac{-1}{2+x} = \dfrac{-1}{2\left(1-\frac{-x}{2}\right)}$

$= \dfrac{-1}{2} \sum_{n=0}^{\infty} \left(\dfrac{-x}{2}\right)^n$

$= \dfrac{-1}{2} \sum_{n=0}^{\infty} \dfrac{(-1)^{n+1} x^n}{2^n}$

$= \sum_{n=0}^{\infty} \dfrac{(-1)^{n+1} x^n}{2^{n+1}}$

R.O.C, $\left|\dfrac{-x}{2}\right| < 1 \Rightarrow |x| < 2 = R$

I.O.C $-2 < x < 2 \Rightarrow (-2, 2)$

$$\boxed{\frac{1}{x-1}} - \frac{1}{x+2} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^{n+1}}} - \sum_{n=0}^{\infty} x^n$$

I.O.C $= (-1, 1)$

R.O.C $= 1$

$$f(x) = \frac{1}{(1+x)^3}$$

$$\Rightarrow \left(\frac{1}{1+x}\right)' = \left(\frac{-1}{(1+x)^2}\right)'$$

$$= -1 \cdot (-2) \cdot \frac{1}{(1+x)^3} = \boxed{2 \cdot \frac{1}{(1+x)^3}}$$

$$\left(\frac{1}{1+x}\right)'' = 2 \cdot \frac{1}{(1+x)^3}$$

$$\Rightarrow \left(\frac{1}{1-(-x)}\right)'' = 2 \cdot \frac{1}{(1+x)^3}$$

$$\Rightarrow \left(\sum_{n=0}^{\infty} (-x)^n\right)''$$

$$\Rightarrow \left(\sum_{n=0}^{\infty} (-1)^n x^n\right)''$$

① $$\Rightarrow \left(\sum_{n=0}^{\infty} n\, x^{n-1} (-1)^n\right)'$$

② $$\Rightarrow \sum_{n=0}^{\infty} n(n-1)\, x^{n-2} (-1)^n = 2 \cdot \frac{1}{(1+x)^3}$$

$$\boxed{\Rightarrow \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n n(n-1) x^{n-2}}$$

R.O.C, $|-x| < 1 = R$

I.O.C $= (-1, 1)$

$(-1, 1)$

$x = -1$, $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n n(n-1)(-1)^{n-2}$

$2 \quad n=0$

$$\Rightarrow \frac{1}{2} \sum_{0}^{\infty} (-1)^{n+n-2} (n)(n-1)$$

$$\Rightarrow \infty$$

$x = 1,\quad \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n n(n-1)$

T.F.D

$(-1, 1)$

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(3) $\rightarrow \sum_{0}^{\infty} \frac{(-1)^n (x-4)^n}{(n+1)^2}, \quad \underline{(3, 5)}$

$x = 3,\quad \sum_{0}^{\infty} \frac{(-1)^n (-1)^n}{(n+1)^2} = \sum_{0}^{\infty} \frac{(-1)^{2n}}{(n+1)^2}$

$$= \sum \frac{1}{(n+1)^2}$$

(cvg)

$x = 5\quad \sum_{0}^{\infty} \frac{(-1)^n}{(n+1)^2}$ A.S.T, $\lim = 0$ decreasing

cvg.

$$\frac{1}{(1+x)^3} = \frac{1}{(1-(-x))^3}$$

$$= \left(\sum (\ )\right)^3$$

के लोकतं ??

$$= \lim_{n\to\infty} \left|\frac{4n^2}{n^2}\right| \cdot \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{2n}$$

$\sum^{\infty} (2n)!\, x^n$

n^{2n}

$$= \lim_{n\to\infty} \left| \frac{(\overbrace{2(n+1)}^{2n+2})!\; x^{n+1}}{(2n)!\; x^{n}} \cdot \frac{n^{2n}}{(n+1)^{2(n+1)}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(2n+2)(2n+1)\cancel{(2n)!}\,\cancel{x^{n}}\cdot|x| \; n^{2n}}{\cancel{(2n)!}\;\cancel{x^{n}} \quad (n+1)^{2n+2}} \right|$$

$$(n+1)^{2n}\cdot(n+1)^{2}$$

$$= \lim_{n\to\infty} \frac{4n^2}{n^2} \cdot \lim_{n\to\infty} \frac{n^{2n}}{(n+1)^{2n}} \quad |x|$$

$$= 4 \cdot \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{2n} |x|$$

$$= 4 \cdot 1/e^{2} \quad |x| < 1$$