$\boxed{\text{NOV 17}}$

I.O.C

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad (-1, 1)$$

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n} \qquad [-1, 1)$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} \qquad (-1, 1]$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad (-\infty, \infty)$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad (-\infty, \infty)$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad (-\infty, \infty)$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad [-1, 1]$$

$$f^n(a) = \underline{n!}\,\underline{c_n}$$

$$c_n = \frac{f^n(a)}{n!}$$

Find series representation of $\ln x$ center at 1

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$\ln x = \ln(1 + x - 1)$$

$$= \ln\left(1 - (-(x-1))\right)$$

$$= -\sum_{n=1}^{\infty} \frac{\left(-(x-1)\right)^n}{n}$$

$$= \underbrace{-}_{(-1)} \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}$$

$$\boxed{= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}}$$

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I.O.C

Ratio Test

$$\lim_{n\to\infty}\left|\frac{(x-1)^{n+1}}{(x-1)^n}\,\frac{n}{(n+1)}\right|$$

$$=\lim_{n\to\infty}\left|\frac{\cancel{(x-1)^n}(x-1)\;n}{\cancel{(x-1)^n}\,(n+1)}\right|$$

$$=|x-1|\lim_{n\to\infty}\left|\frac{n}{n+1}\right|$$

$$=|x-1|<1=R$$

$$-1<x-1<1$$

$$0<x<2$$

Check, $x=0$, $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n}{n}$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{2n+1}}{n}$$

$$=-\sum_{n=1}^{\infty}\frac{1}{n}\quad(\text{DIV})$$

$x=2$, $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\,1^n}{n}$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\qquad \text{A.S.T}$$

$=$ converges by A.S.T

$$\text{I.O.C}=(0,2]$$

Find the sum of the series.

$$\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{6^{2n}(2n)!}\qquad \left|\;\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}\right.$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n(\pi/6)^{2n}}{(2n)!}\qquad x=\pi/6$$

$$(2n)!$$

$$= \cos(\pi/6) \boxed{= \sqrt{3}/2}$$

Find the 10^{th} derivative of $f(x) = \arctan\left(x^2/3\right)$ at $x=0$

$$f^{10}(0)\ ??$$

$$f^{n}(a) = n!\, c_n$$

$$f^{10}(0) = 10! \cdot c_{10}$$

$$f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + \underline{c_2} x^{2} + \underline{c_3} x^{3} + \dots$$

$$\dots + \underline{c_{10}} x^{10} + \dots$$

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)}$$

$$\arctan\left(x^2/3\right) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2(2n+1)}}{3^{2n+1}(2n+1)} \leftarrow$$

$$= \sum_{n=0}^{\infty} \underbrace{\frac{(-1)^n}{3^{2n+1}(2n+1)}}_{c_{10} \text{ when } n=2} x^{4n+2}$$

$$4n+2 = 10$$

$$4n = 8$$

$$n = 2$$

$$c_{10} = \frac{(-1)^2}{3^{2\cdot 2+1} \cdot (2\cdot 2+1)}$$

$$\boxed{c_{10} = \frac{1}{3^5 \cdot 5}}$$

$$f^{10}(0) = 10!\, c_{10}$$

$$\boxed{= 10! \cdot \frac{1}{3^5 \cdot 5}}$$

Use $\cos(x^4)$ to evaluate $\leftarrow$

$$\lim_{x\to 0} \frac{\cos(x^4) - 1 + x^8/2}{x^{16}}$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos(x^4) = \sum_{n=0} \frac{(-1)^n x^{4(2n)}}{(2n)!}$$

$$\cos(x^4) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{8n}}{(2n)!}$$

$$\frac{\cos(x^4) - 1 + \frac{x^8}{2}}{x^{16}} = \frac{\cancel{1}_{n=0} - \cancel{\frac{x^8}{2}}_{n=1} + \frac{x^{16}}{4!}_{n=2} + \sum_{n=3}^{\infty} \frac{(-1)^n x^{8n}}{(2n)!} \cancel{-1} + \cancel{\frac{x^8}{2}}}{x^{16}}$$

$$\frac{\cos(x^4) - 1 + x^8/2}{x^{16}} = \underbrace{\frac{\cancel{x^{16}}}{4!\,\cancel{x^{16}}}}_{n=2} - \underbrace{\frac{x^{24}}{6!\,x^{16}}}_{n=3} + \underbrace{\frac{x^{32}}{8!\,x^{16}}}_{n=4} + \cdots$$

$$= \frac{1}{24} - \frac{x^{24-16}}{6!} + \frac{x^{32-16}}{8!} + \cdots$$

$$\lim_{x \to 0} \frac{\cos(x^4) - 1 + x^8/2}{x^{16}} = \frac{1}{24} - \underbrace{\frac{x^8}{6!}}_{0} + \underbrace{\frac{x^{16}}{8!}}_{0} + \underbrace{\cdots}_{0}$$

$$\boxed{= \frac{1}{24}}$$

$\cos(1/2)$ (Find two nonzero term & find the error)

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$= \underset{n=0}{1} - \underset{n=1}{\frac{x^2}{2}}$$

$$\cos(1/2) = 1 - \frac{(1/2)^2}{2}$$

$$= 1 - \frac{1}{8}$$

$$|\text{error}| < b_{N+1}$$

$$< b_2 = \frac{x^{2 \cdot 2}}{(2 \cdot 2)!}$$

$$x^4$$

$= \frac{x}{4!}$

$= \left(\frac{1}{2}\right)^4 \cdot \frac{1}{24}$

$= \frac{1}{16} \cdot \frac{1}{24}$

$\boxed{= \frac{1}{384}}$

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e^x centre at 0

A $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \longleftrightarrow e^{2 \cdot 0 \cdot 1}$

Find error

$= \_ \; \_ \; \_ \; \_ \; \_ \; \_$

B e^x centre at 2

$e^x = \sum_{n=0}^{\infty} \frac{(x-2)^n}{} \longleftrightarrow e^{2 \cdot 0 \cdot 1}$

Find error.

$= \sim \; \sim \; \sim$