Oct 13

Hierarchy of function:

$\ln(n) << n^a << a^n << n! << n^n$
$(a>0)$ $(a>1)$

If given rational function or sequence.
Let L is given limit as $n \to \infty$

① If deg (numerator) > deg (denominator)
then $L = \pm\infty$ (Diverges)

② If deg (numerator) = deg (denominator)
then L = ratio of leading coefficient

③ If deg (numerator) < deg (denominator)
then $L = 0$

Increasing : A sequence a_n is said to be increasing if for every n $a_{n+1} \geq a_n$

Decreasing — — —
— — — — . $a_{n+1} \leq a_n$

Monotonic If a sequence is either increasing or decreasing then it is monotonic

Bounded

A sequence is said to be bounded

If the sequence eventually goes to finite term.

1) $\lim_{n\to\infty} (-1)^n = \{-1, 1, -1, 1 \ldots\}$ ~~Div~~

limit DNE

2) $\lim_{n\to\infty} \sin\left(\frac{n\pi}{2}\right) = \{1, 0, -1, 0, 1 \ldots\}$ ~~Div~~

limit DNE

3) $\lim_{n\to\infty} \cos(2n\pi) = \{1, 1, 1 \ldots\}$ $\underline{1}$

~~Converge~~

4) $\lim_{n\to\infty} \frac{(n-1)!}{(n+1)!}$

$5! = 5\cdot\underbrace{4\cdot 3\cdot 2\cdot 1}$

$3!$

$= \lim_{n\to\infty} \frac{\cancel{(n-1)!}}{(n+1)\cdot n\cdot \cancel{(n-1)!}}$

$= \lim_{n\to\infty} \frac{1}{(n+1)\cdot n}$ $\boxed{= 0 \text{ converges}}$

$a_n = \left(1+\frac{1}{n}\right)^n = f(x) = \left(1+\frac{1}{x}\right)^x$

$\Rightarrow \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$

$\Rightarrow \lim_{n\to\infty} e^{\ln\left(1+\frac{1}{n}\right)^n}$

$\Rightarrow \lim_{n\to\infty} e^{n\ln\left(1+\frac{1}{n}\right)}$

$\Rightarrow e^{\lim_{n\to\infty} n\ln\left(1+\frac{1}{n}\right)}$ $(\infty\cdot 0)$

$\Rightarrow \lim \frac{\ln\left(1+\frac{1}{n}\right)}{\ldots}$ $\left(\frac{0}{\ldots}\right)$

$$\Rightarrow e^{\lim_{n\to\infty} \frac{\ \ }{\frac{1}{n}}} \quad \left(\frac{0}{0}\right)$$

$$\Rightarrow e^{\lim_{n\to\infty} \frac{\frac{1}{1+\frac{1}{n}} \cdot \cancel{\frac{-1}{n^2}}}{\cancel{\frac{-1}{n^2}}}}$$

$$\Rightarrow e^{\lim_{n\to\infty} \frac{1}{1+\frac{1}{n}}}$$

$$= e^{1} \quad \boxed{= e}$$

(6) $\left\{ \frac{\ln(1000000\,n)}{2^n} \right\} \to$ converges

(7) $\left\{ \frac{2^n}{3^{n+1}} \right\} \longrightarrow$ converge to 0

$\longrightarrow \frac{2^n}{3^n \cdot 3}$

$$\frac{2}{3^2} \quad \frac{2^2}{3^3} \quad \frac{2^3}{3^4} \quad \cdots \Rightarrow \frac{1}{3}\left(\frac{2}{3}\right)^n$$

$$\frac{2}{3} \qquad \frac{2}{3} \qquad \frac{2}{3}$$

$r = \frac{2}{3}$ $\qquad |r| < 1$

$\boxed{\text{converges to } 0}$

(9) $\left\{ \frac{5^n}{n^5} \right\} \longrightarrow$ Diverges

(10) $\left\{ \frac{10n^3 + 15n^2 + 12n + 5}{2n^3} \right\}$

$\Rightarrow \frac{10}{\ }$ $\boxed{= 5}$ — converges

converges

(11) $a_n = \left(\frac{e}{\pi}\right)^n$

$e = 2.76\ldots$
$\pi = 3.14\ldots$

$$\frac{e}{\pi} \xrightarrow{\cdot \frac{e}{\pi}} \frac{e^2}{\pi^2} \xrightarrow{\cdot \frac{e}{\pi}} \frac{e^3}{\pi^3} \xrightarrow{\cdot \frac{e}{\pi}} \frac{e^4}{\pi^4}$$

$a_n > a_{n+1}$

(1) Convergent
(2) Bounded
(3) Geometric
(4) Decreasing
(5) Monotonic

If $a_n \le b_n \le c_n$ and for $(n \ge N)$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then

$$\lim_{n\to\infty} b_n = L$$

$\left\{\frac{n!}{n^n}\right\} \to$ converges to 0

$n^n \gg n!$

positive $\to$ $\frac{n!}{n^n}$ $\leftarrow$ Positive

$$\frac{n!}{n^n} = \underbrace{\frac{n}{n} \cdot \frac{n-1}{n} \cdot \frac{n-2}{n} \cdots \frac{2}{n}}_{1 \quad <1 \quad <1 \quad \cdots \quad <1} \cdot \frac{1}{n}$$

$$\frac{5!}{5^5} = \frac{5}{5}\cdot\frac{4}{5}\cdot\frac{3}{5}\cdot\frac{2}{5}\cdot\frac{1}{5}$$

$$< 1 \cdot \frac{1}{n}$$

$$0 \le \frac{n!}{n^n} \le \frac{1}{n}$$

$$\lim_{n\to\infty} 0 = 0 \qquad \boxed{0} \qquad \lim_{n\to\infty} \frac{1}{n} = 0$$

$$1 + (-1)^n$$

$\Rightarrow$ Diverges limit DNE

Bounded $-1 < 1+(-1) < 3$