Oct 20

Dictionary

Series : $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_N + a_{N+1} + \ldots$

S_1

S_2

S_3

$\ldots$

S_N

$$\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_N = S$$

finite (CONVERGE)

Infinite (DIVERGE)

. . .

	SEQUENCE	SERIES
	$\{a_n\}$ $\lim_{n\to\infty} a_n = L < \infty$	$\sum a_n$ $\lim_{N\to\infty} S_N = S < \infty$
$\lim a_n = 0$	Converges to 0	? (No conclusion)
$\lim a_n = c \neq 0 < \infty$	converges to c	Div (TFD)
[illegible]		Converges to [illegible]

$\lim S_n = 0$		Converges to 0
$\lim S_n = C \neq 0 < \infty$		Converges to c

Geometric Series

$$\sum_{n=1}^{\infty} ar^n = \underline{ar} + \underline{ar^2} + \underline{ar^3} + \dots + ar^n$$

$$= \frac{ar}{1-r} \quad \text{if } |r| < 1$$

$ar \to$ 1st term

(Diverges if $|r| \geq 1$)

Telescopic Series

1. Find PFD
2. Compute S_N
3. If $\lim_{N \to \infty} S_N = c$ (finite then it converges)

(If not, diverges)

Test For Divergence (TFD)

If $\lim_{n \to \infty} a_n \neq 0$, then the series $\sum_{n=0}^{\infty} a_n$ diverges.

Integral Test

If 'f' is positive, continuous and (eventually) decreasing over $[1, \infty)$ and $a_n = f(n)$, then

$\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either both

converge or diverge.

P TEST

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge for $p > 1$ and
diverge for $p \leq 1$

GENERALISED P TEST

$\sum_{n=2}^{\infty} \frac{1}{n^p (\ln(n))^q}$ converge if $p > 1$
diverge if $p < 1$

and,
if $p = 1$, it converge if $q > 1$ and
diverge if $q \leq 1$

(*) Only two type of series tells you exact sum, when it converges i.e (Geometric and Telescopic Series)