Oct 20

$\circledast$ $0.\overline{2312}$ $\qquad \sum_{n=1}^{\infty} \frac{2312}{10000^n}$

$a_1 = \frac{2312}{10000} = 0.2312 = S_1$

$a_2 = \frac{2312}{(10000)^2} = \frac{0.2312}{(10000)(10000)} = 0.00002312$

$S_2 = a_1 + a_2$

$= 0.2312 + 0.00002312$

$= 0.2312\,2312$

$S_3 = 0.2312\,2312\,2312$

$S_n = 0.2 \ldots$

$\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$

$= \ln\left(\frac{n^2-1}{n^2}\right)$

$= \ln\left(\frac{(n-1)(n+1)}{n^2}\right)$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$= \ln\big((n-1)(n+1)\big) - \ln n^2$

$\ln(ab) = \ln a + \ln b$

$\ln a^n = n \ln a$

$= \ln(n-1) + \ln(n+1) - 2\ln n$

$$= 1\ln(n-1) - 2\ln n + 1\ln(n+1)$$

$$= \ln(n-1) - \ln(n) - \ln(n) + \ln(n+1)$$

$(n=2)$ $\ln(1) - \cancel{\ln(2)} - \ln(2) + \cancel{\ln(3)}$

$(n=3)$ $\cancel{\ln(2)} - \cancel{\ln(3)} - \cancel{\ln(3)} + \cancel{\ln(4)}$

$(n=4)$ $\cancel{\ln(3)} - \cancel{\ln(4)} - \cancel{\ln(4)} + \cancel{\ln(5)}$

$(n=5)$ $\cancel{\ln(4)} - \cancel{\ln(5)} - \cancel{\ln(5)} + \cancel{\ln(6)}$

$\vdots$

$\cancel{\ln(N-2)} - \cancel{\ln(N-1)} - \cancel{\ln(N-1)} + \cancel{\ln(N)}$

$(n=N)$ $\cancel{\ln(N-1)} - \ln(N) - \cancel{\ln(N)} + \ln(N+1)$

$$S_N = \underbrace{\ln(1)}_{0} - \ln(2) + \ln(N+1) - \ln(N)$$

$$\sum_{n=2}^{\infty} a_n = \lim_{N\to\infty} S_N$$

$$= \lim_{N\to\infty} -\ln 2 + \ln(N+1) - \ln(N)$$

$$(\infty - \infty)$$

$$= -\ln 2 + \lim_{N\to\infty} \ln\left(\frac{N+1}{N}\right)$$

$$= -\ln 2 + \lim_{N\to\infty} \ln\left(\frac{N}{N} + \frac{1}{N}\right) = \ln\left(1 + \frac{1}{N}\right)$$

$$\ln\left(1 + \underbrace{\frac{1}{\infty}}_{0}\right)$$

$$\ln(1)$$

$$= -\ln 2 + \underbrace{\ln 1}_{0}$$

$$\boxed{= -\ln 2}$$

For what values of p would the series converge?

$$\sum_{n=3}^{\infty} \frac{1}{n \ln n [\ln(\ln n)]^p} = \int_3^{\infty} \frac{1}{x \ln x (\ln(\ln x))^p} dx$$

u

$$\frac{1}{n^p [\ln n]^q}$$

$u = \ln(\ln n)$

$$du = \frac{1}{\ln \cancel{x}} \cdot \frac{1}{\cancel{x}} dx$$

$$\int_3^{\infty} \frac{1}{u^p} du$$

, for $p > 1$ converge

$p \leq 1$ (Diverge)

$$\frac{1}{n^{0.2} [\ln n]^{500}} \quad (\text{Div})$$

$$\frac{1}{\sqrt{n^3} [\ln(n)]^{0.3}} \quad (\text{Conv})$$

$n^{3/2} = 1.5$

$$\frac{1}{n [\ln n]^{500}}$$

$\xrightarrow{}$ converge / $q > 1$

$$\sum^{\infty} \frac{\ln n}{} \leftarrow \ln \quad \int^{\infty} \frac{\ln x^2}{} dx$$

$n=2$ $\quad n \Leftarrow$ inc $\quad 2 \quad \frac{x}{\checkmark} \Rightarrow 2$

I.T

1. positive ✓
2. Continuous ✓
3. Decreasing ✓

$\left[\frac{n}{n+1} \nearrow \quad \frac{n+1}{n} \searrow\right]$

$\frac{1}{n^2} \quad \Big| \quad \frac{n}{3}$

$f'(x) < 0$

$$f'(x) = \frac{\cancel{x} \cdot \frac{1}{\cancel{x}} - \ln x \cdot 1}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

$x^2 = +ve$

$1 - \ln x < 0$

$1 < \ln x$

$e < e^{\ln x}$

$2.71 \quad \boxed{e < x}$

$\sum_{n=2}^{\infty} \frac{\ln n}{n}$

No.

$$\int_3^{\infty} \frac{\ln x}{x}\,dx$$

u = sub
$u = \ln x$
$du = \frac{1}{x}\,dx$

$$\lim_{t \to \infty} \int_3^{t} \frac{\ln x}{x}\,dx$$

$= \lim \int u\,du$

$t \to \infty$

$$= \lim_{t\to\infty} \left.\frac{(\ln x)^2}{2}\right|_3^t$$

$$= \lim_{t\to\infty} \frac{(\ln t)^2}{2} - \frac{(\ln 3)^2}{2}$$

$$= \infty \quad (\text{Div})$$

$$\sum_{n=2}^{\infty} \frac{\ln n}{n} = \frac{\ln 2}{2} + \sum_{n=3}^{\infty} \frac{\ln n}{n}$$

$$= \frac{\ln 2}{2} + \infty$$