September 22
(L5 – L7)

$\sqrt{a^2-x^2}$, let $x = a\sin\theta$, $\sqrt{a^2-x^2} = a\cos\theta$

$\sqrt{x^2+a^2}$, let $x = a\tan\theta$, $\sqrt{x^2+a^2} = a\sec\theta$

$\sqrt{x^2-a^2}$, let $x = a\sec\theta$, $\sqrt{x^2-a^2} = a\tan\theta$

$$\sqrt{a^2-x^2}$$
$$= \sqrt{a^2 - a^2\sin^2\theta}$$
$$= \sqrt{a^2(1-\sin^2\theta)}$$
$$= \sqrt{a^2\cos^2\theta} = a\cos\theta$$

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx = \int \frac{e^{2x}\cdot e^{x}\,dx}{1+e^{2x}}$$

$u = e^x$

$du = e^x\,dx$

$$\int \frac{u^2\,du}{1+u^2}$$
$$= \int \frac{(1+u^2)-1}{1+u^2}\,du$$
$$= \int \left(\frac{1+u^2}{1+u^2} - \frac{1}{1+u^2}\right)du$$
$$= \int \left(1 - \frac{1}{1+u^2}\right)du$$
$$= u - \arctan(u)$$
$$\boxed{= e^x - \arctan(e^x) + C}$$

$$\int \frac{dx}{x\sqrt{64x^4-1}} = \int \frac{\cancel{u}\cdot du}{128x^3\cdot x\,\cancel{u}}$$

$$u = \sqrt{64x^4 - 1}$$

$$du = \frac{1}{\cancel{2}\sqrt{64x^4-1}} \cdot \cancel{4}^{2} \cdot 64x^3\,dx$$

$$= \frac{1}{u} \cdot 128x^3\,dx$$

$$\frac{u \cdot du}{128x^3} = dx$$

$$\int \frac{1}{128x^3 \cdot x}\,du$$

$$= \int \frac{1}{128x^4}\,du$$

$$= \int \frac{1}{\cancel{128}\,2\,\frac{1+u^2}{\cancel{64}}}\,du$$

$$= \frac{1}{2}\int \frac{1}{1+u^2}\,du$$

$$= \frac{1}{2}\arctan(u)$$

$$\boxed{= \frac{1}{2}\arctan\left(\sqrt{64x^4-1}\right) + C}$$

$$u = \sqrt{64x^4-1}$$

$$u^2 = 64x^4 - 1$$

$$1 + u^2 = 64x^4$$

$$\frac{1+u^2}{64} = x^4$$

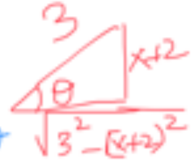
$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx$$

$$= \int \frac{1}{(-(x^2+4x-5))^{5/2}}\,dx$$

$$= \int \frac{1}{(-(x^2+4x+4-5-4))^{5/2}}\,dx$$

$$= \int \frac{1}{(-((x+2)^2 - 3^2))^{5/2}}\,dx$$

$$= \int \frac{1}{(3^2-(x+2)^2)^{5/2}}\,dx$$



$$x + 2 = 3\sin\theta$$

$$dx = 3\cos\theta\,d\theta$$

$$\int \frac{1}{\quad} \; 3\cos\theta\,d\theta$$

$$= \int \frac{}{(3^2 - 3^2\sin^2\theta)^{5/2}}$$

$$= \int \frac{3\cos\theta \, d\theta}{(3^2 \cos^2\theta)^{5/2}}$$

$$= \int \frac{\cancel{3\cos\theta}}{\cancel{(3\cos\theta)^{2 \cdot 5/2}}} d\theta \quad = \frac{\cancel{3\cos\theta}}{(3\cos\theta)^{\cancel{5}\,4}}$$

$$= \int \frac{1}{(3\cos\theta)^4} d\theta$$

$$= \frac{1}{81} \int \frac{1}{\cos^4\theta} d\theta$$

$$= \frac{1}{81} \int \sec^4\theta \, d\theta$$

$$= \frac{1}{81} \int \sec^2\theta \sec^2\theta \, d\theta$$

$$= \frac{1}{81} \int (1+\tan^2\theta)\sec^2\theta \, d\theta$$

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$$= \frac{1}{81} \int (1+u^2) \, du$$

$$= \frac{1}{81}\left(u + \frac{u^3}{3}\right)$$

$$= \frac{1}{81}\left(\tan\theta + \frac{\tan^3\theta}{3}\right)$$

$$\boxed{= \frac{1}{81}\left(\frac{x+2}{\sqrt{3^2-(x+2)^2}} + \frac{1}{3}\left(\frac{x+2}{\sqrt{3^2-(x+2)^2}}\right)^3\right) + C}$$

$$\int \frac{1}{(x+5)^2(x-1)} dx$$

$$= \frac{A}{(x+5)^2} + \frac{B}{(x+5)} + \frac{C}{(x-1)}$$

x^2+5

$\frac{Ax+B}{x^2+5}$

$x^2+5 \neq (x+5)^2$

$(x+5)(x+5)$

$$\boxed{\frac{A}{(x+5)^2}+\frac{B(x+5)}{(x+5)}+\frac{C}{(x-1)}=\frac{1}{(x+5)^2(x-1)}}$$

Cover up

$A+B+C$

Let $x=-5$, $A=\frac{1}{(-5-1)}$

$=-\frac{1}{6}$

$x=1$, $C=\frac{1}{(1+5)^2}=\frac{1}{36}$

$x=0$

$$\frac{-1}{6\cdot 25}+\frac{B}{5}+\frac{1}{36(-1)}=\frac{1}{25\cdot(-1)}$$

$$\frac{B}{5}=\frac{1}{6\cdot 25}+\frac{1}{36}-\frac{1}{25}$$

$$\frac{B}{5}=\frac{6+25-36}{36\cdot 25}$$

$$\frac{B}{\cancel{5}}=\frac{\cancel{-5}}{36\cdot \cancel{25}}\ 5$$

$$\boxed{B=\frac{-1}{36}}$$

$$\boxed{A=\frac{-1}{6},\ B=\frac{-1}{36},\ C=\frac{1}{36}}$$

$$\int\frac{-1}{6(x+5)^2}dx+\int\frac{-1}{36(x+5)}dx+\int\frac{1}{36(x-1)}dx$$

$$=\frac{1}{6(x+5)}\ \frac{-1}{36}\ln|x+5|+\frac{1}{36}\ln|x-1|$$

$$= \frac{1}{6(x+5)} + \frac{1}{36}\left(\ln|x-1| - \ln|x+5|\right)$$

$$\boxed{= \frac{1}{6(x+5)} + \frac{1}{36}\left(\ln\left|\frac{x-1}{x+5}\right|\right) + C}$$

$$\int \frac{-1}{6(x+5)^2}\,dx$$

let $u = x+5$

$du = dx$

$$-\frac{1}{6}\int \frac{1}{u^2}\,du$$

$$= -\frac{1}{6}\,\frac{u^{-2+1}}{(-2+1)}$$

$$= \cancel{-}\frac{1}{6}\,\frac{u^{-1}}{\cancel{(-1)}}$$