September 24

$$\int_0^1 \frac{1}{1+x^{1/3}}\,dx$$

u-sub

$u = x^{1/3}+1$

$du = \frac{1}{3x^{2/3}}\,dx$

$3u^2\,du = dx$

$$= \int_0^1 \frac{3u^{\cancel{2}}\,du}{\cancel{u}}$$

$$= 3\frac{u^2}{2}\Big|$$

$$= \frac{3}{2}\left(x^{1/3}+1\right)^2\Big|_0^1$$

$$= \frac{3}{2}\left(x^{2/3}+2x^{1/3}+1\right)\Big|_0^1$$

$$= \frac{3}{2}(1+2+1) - \frac{3}{2}(0+0+1)$$

$$= \frac{3}{\cancel{2}}\cdot\cancel{4}^{2} - \frac{3}{2} = 6-\frac{3}{2}$$

$$\boxed{=\frac{9}{2}}$$

$u = x^{1/3}$

$= 3\ln 2 - \frac{3}{2}$

$$\int_0^1 \frac{1}{1+u}\,3u^2\,du$$

$u = x^{1/3}$

$du = \frac{1}{3x^{2/3}}\,dx$

$du = \frac{1}{3u^2}\,dx$

$3u^2\,du = dx$

$$= 3\int \frac{u^2}{u+1}\,du$$

$$= 3\int\left(u-1+\frac{1}{u+1}\right)du$$

$$u+1\,\overline{)\,u^2\,}\quad u-1$$
$$-(u^2+u)$$
$$-u$$
$$-(-u-1)$$
$$1$$

$$= 3\left(\frac{u^2}{2} - u + \ln|u+1|\right)$$

$$= 3\left(\frac{x^{2/3}}{2} - x^{1/3} + \ln\left|x^{1/3}+1\right|\right)\Big|^1$$

$$= 3\left(\frac{1}{2} - 1 + \ln 2\right) - 3\underbrace{\left(0 - 0 + 0\right)}_{0}$$

$$\boxed{= 3\left(-\frac{1}{2} + \ln 2\right)}$$

$$\boxed{= 3\left(\ln 2 - \frac{1}{2}\right)}$$

$\int_1^\infty \frac{1}{x^p}\,dx$ is <u>convergent</u> for $p > 1$ and <u>Divergent</u> for $p \le 1$

$$\int_1^\infty \frac{1}{x^{1.00000001}}\,dx$$

Convergent

Type 1

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{t\to\infty} \int_t^b f(x)\,dx$$

$$\int_0^\infty \frac{e^x}{e^{2x}+3}\,dx$$

$$= \lim_{t\to\infty} \int_0^t \frac{e^x}{e^{2x}+3}\,dx$$

$u = e^x$

$du = e^x\,dx$

$$= \lim_{t\to\infty} \int \frac{du}{u^2+3}$$

$$= \lim_{t\to\infty} \frac{1}{\sqrt{3}} \arctan\left(\frac{u}{\sqrt{3}}\right)$$

$$= \lim_{t\to\infty} \frac{1}{\sqrt{3}} \arctan\left(\frac{e^x}{\sqrt{3}}\right)\Bigg|_0^t$$

$$= \frac{1}{\sqrt{3}}\left(\lim_{t\to\infty} \arctan\left(\frac{t}{\sqrt{3}}\right) - \lim_{t\to\infty} \arctan\left(\frac{c^0}{\sqrt{3}}\right)\right)$$

$$= \frac{1}{\sqrt{3}}\left(\arctan(\infty) - \arctan\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$\boxed{= \frac{1}{\sqrt{3}} \cdot \frac{\pi}{3}} \text{ OR } \boxed{\frac{\sqrt{3}\pi}{9}}$$

Type 2

(a) If "f" is cts on $[a, b)$ and is dis cts at b, then

$$\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$$

(b) If "f" is cts on $(a, b]$ and is discts at a, then

$$\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$$

$$\int_0^{\pi/4} \csc^2(\theta) - \csc(\theta)\cot(\theta)d\theta$$

$\infty \quad -\infty$

$$= \lim_{t\to 0^+} \int_t^{\pi/4} \csc^2(\theta) - \csc(\theta)\cot(\theta)\, d\theta$$

$$= \lim_{t\to 0^+}\left(-\cot(\theta) + \csc(\theta)\right)\Big|_t^{\pi/4}$$

$$= \lim_{t\to 0^+}\left(\csc\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right)\right) - \lim_{t\to 0^+}\left(\csc(t) - \cot(t)\right)$$

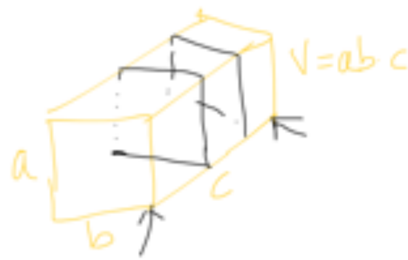
$$= \left(\sqrt{2} - 1\right) - \lim_{t\to 0^+}\left(\frac{1}{\sin t} - \frac{\cos t}{\sin t}\right)$$

$$= (\sqrt{2} - 1) - \lim_{t\to 0^+}\left(\frac{1-\cos t}{\sin t}\right) \quad \left(\frac{0}{0}\right)$$

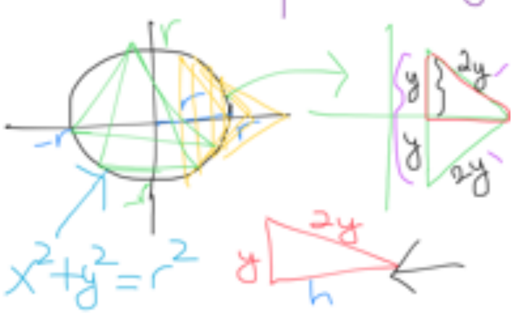
$$= (\sqrt{2}-1) - \lim_{t\to 0^+} \frac{\sin t}{\cos t} \to \tan t$$

$$= (\sqrt{2}-1) - \underbrace{\tan 0}_{0}$$

$$\boxed{= \sqrt{2} - 1}$$



Find the volume of the solid whose base is a disk of radius r and whose cross-sections are equilateral triangle



$$h = \sqrt{(2y)^2 - y^2}$$

$$= \sqrt{4y^2 - y^2} = \sqrt{3y^2}$$

$$= \sqrt{3}\, y$$

Area of $\Delta = \frac{1}{2} b \cdot h$

$$= \frac{1}{\cancel{2}} \cancel{2}y \cdot \sqrt{3}\, y$$

$$= \sqrt{3}\, y^2$$

$$V = \int^{r}$$

$$V = \int_{-r} \sqrt{3}\, y\, dx \qquad x^2+y^2=r^2$$

$$y^2=r^2-x^2$$

$$= \int_{-r}^{r} \sqrt{3}\,(r^2-x^2)\,dx$$

$$= \sqrt{3}\left(r^2x - \frac{x^3}{3}\right)\Big|_{-r}^{r}$$

$$= \sqrt{3}\left(r^2\cdot r - \frac{r^3}{3} - \left(-r^3 + \frac{r^3}{3}\right)\right)$$

$$= \sqrt{3}\left(r^3 - \frac{r^3}{3} + r^3 - \frac{r^3}{3}\right)$$

$$= \sqrt{3}\left(2r^3 - \frac{2r^3}{3}\right)$$

$$\boxed{= \sqrt{3}\left(\frac{4r^3}{3}\right)}$$