ABSTRACTS FOR OTWIA 2020

1. Plenary talks

Speaker: Raphaël Clouâtre (University of Manitoba)

Title: Finite-dimensional approximations on the bidisc

Abstract: Norms of multipliers of reproducing kernel Hilbert spaces are completely determined by finite-dimensional information. More precisely, multiplier algebras are "residually finite-dimensional". In this talk, we will discuss connections between the abundance of their finite-dimensional representations and the structure of some associated universal C*-algebras. Mysteries arise even in the familiar setting of the bidisc algebra. This is joint work in progress with Adam Dor-On.

Speaker: Stephan R. Garcia (Pomona College)

Title: An Operator Theorist does Combinatorics: Numerical Semigroups and Positivity

Abstract: How did an operator theorist get involved in combinatorics? How do the two fields interact? Using tools from complex, harmonic, and functional analysis, probability theory, algebraic combinatorics, and computer-aided design, we answer virtually all asymptotic questions about factorization lengths in numerical semigroups. This yields uncannily accurate predictions that agree with numerical computations. We also present positivity results for certain multivariate polynomials, potential applications to AF algebras, and possible generalizations via several complex variables.

Speaker: Pamela Gorkin (Bucknell University and NSF)

Title: Blaschke Products and the Curve of Geodesic Centers

Abstract: Let B be a Blaschke product of degree three with zeros a, b and 0. For each point w on the unit circle, if you form the triangle with vertices at the three points B maps to w, that triangle will circumscribe the ellipse with foci at a and b and minor axis $\sqrt{(1-|a|^2)(1-|b|^2)}$. There is a generalization of this result for Blaschke products of degree n. The work that will be discussed in this talk revisits these results, but this time we consider the Poincaré model. This will be based on joint work with U. Daepp, G. Semmler, and E. Wegert as well as recent work with G. Adams.

Speaker: Irina Holmes (Texas A&M University)

Title: Bi-parameter embeddings on dyadic bi-trees

Abstract: This work is part of recent investigations of analysis on trees and bi-trees, motivated by characterizing Carleson measures for the Dirichlet space on the bidisc. Roughly speaking, the connection between the two comes from identifying the unit disc (or bi-disc) in the complex plane with the dyadic tree (or bi-tree) indexing its Whitney decomposition. A recent Carleson embedding theorem (joint with Arcozzi, Mozolyako and Volberg) will be discussed, as well as a new line of investigation for bi-trees similar to the "product BMO vs rectangular BMO" in multi-parameter analysis — a famous counterexample by Carleson.

Speaker: John M$^{\text{c}}$Carthy (Washington University in St. Louis)

Title: Retracting retracts

Abstract: If U is a domain in $\mathbb{C}^n$ and $V \subseteq U$, we say that (U, V) has the isometric extension property if every bounded holomorphic function on V extends to a function of the same norm in $H^\infty(U)$. If V is a retract of U, then (U, V) always has the isometric extension property. Various results over the past two decades have proved a converse of this, under certain hypotheses on U and V.

We shall review the history of these results, and why they were studied in the first place. Then we shall discuss whether the emphasis on retracts is misplaced.

This is joint work with Jim Agler and Lukasz Kosinski.

Speaker: Scott McCullough (University of Florida)

Title: Convexity in Free Analysis

Abstract: Noncommutative analysis, also known as free analysis, is the study of analytic aspects of algebra in freely noncommuting variables. It includes the skew field of free rational functions, free function theory and free semialgebraic geometry, that latter two subjects developed by analogy to their classical commutative counterparts. It has connections to free probability; operator algebras/systems/spaces and completely positive maps; several complex variables; and applications in system engineering. Indeed, many systems engineering problems convert, rather mechanically, to (systems of) matrix inequalities. Mathematically these matrix inequalities are modeled as the set of tuples of matrices on which a matrix-valued free polynomial, or more generally free rational function, takes positive semidefinite values; that is, the a free analog of a semialgebraic set. For optimization purposes, convexity of these sets is often desired. This talk will survey a sampling of results in free analysis with an emphasis on convexity.

Speaker: Hugo J. Woerdeman (Drexel University)

Title: 2TALK: Complete spectral sets and numerical range & Maximum determinant positive definite Toeplitz completions.

Abstract: Part 1: We define the complete numerical radius norm for homomorphisms from any operator algebra into $B(H)$, and show that this norm can be computed explicitly in terms of the completely bounded norm. This is used to show that if K is a complete C-spectral set for an operator T, then it is a complete M-numerical radius set, where $M = (C+1/C)/2$. This also leads to variations of Crouzeix's conjecture, where the operator norm is replaced by the numerical radius. This is joint work with Kenneth Davidson and Vern Paulsen.

Part 2: We consider partial symmetric Toeplitz matrices where a positive definite completion exists. We characterize those patterns where the maximum determinant completion is itself Toeplitz. We then extend these results with positive definite replaced by positive semidefinite, and maximum determinant replaced by maximum rank. These results are used to determine the singularity degree of a family of semidefinite optimization problems. This is joint work with Stefan Sremac and Henry Wolkowicz.

2. Semi-plenary talks

Speaker: Meric Augat (Washington University in St. Louis)

Title: Differentiation and antidifferentiation in free analysis

Abstract: In recent years, several classical theorems about analytic functions have been extended to the setting of free analysis e.g. the inverse function theorem has been generalized to matrix and operator settings. In particular, the free (matrix) inverse function theorem has a stronger statement than its classical counterpart, partly due a remarkable property of differentiation in free analysis: the free derivative can be realized via point evaluation.

In this talk, we first discuss the free derivative, its remarkable properties and their consequences. Next, we investigate necessary and sufficient conditions for antidifferentiation of free functions. In fact, we generalize two classical theorems to free analysis; if F is a differentiable vector field equal to the gradient of some potential function, then F is curl free and conversely, if F is a curl free vector field on a simply connected domain, then F is the derivative of a potential function.

Our two main results are as follows: the derivative of an analytic free map must be free-curl free and when we are on a connected free domain, every free-curl free map can be antidifferentiated.

Speaker: Nikolaos Chalmoukis (University of Bologna)

Title: Random Interpolating Sequences in Dirichlet Type Spaces

Abstract: In the context of the standard weighted Dirichlet spaces in the unit disc we will discuss a random version of the classical interpolation problem. In particular we consider Steinhaus random variables and we derive the Kolmogorov 0-1 law for universal interpolation, weak separation and the

Carleson measure condition. Our results generalize and improve upon earlier work of Rudowicz and Bogdan and Cochran. This is a joint work with A. Hartman, K. Kellay and B. Wick.

Speaker: Chao Ding (Masaryk University)

Title: Boundary value problems in Euclidean space for bosonic Laplacians

Abstract: A bosonic Laplacian is a conformally invariant second order differential operator acting on smooth functions defined on domains in Euclidean space and taking values in higher order irreducible representations of the special orthogonal group. In this talk, we introduce boundary value problems involving bosonic Laplacians in the upper-half space and the unit ball. We also show the uniqueness for solutions to the Dirichlet problems with continuous data for bosonic Laplacians and provide analogs of some properties of harmonic functions for null solutions of bosonic Laplacians, for instance, Cauchy's estimates, the mean-value property, Liouville's Theorem, etc. This is a joint work with Phuoc-Tai Nguyen and John Ryan.

Speaker: Christopher Felder (Washington University in St. Louis)

Title: (General) Optimal Polynomial Approximants

Abstract: We will first situate ourselves in H– a suitably 'nice' Hilbert space of functions on the unit disk– and then dive into the study of optimal polynomial approximants. That is, fixing $f \in H$ and varying $n \in \mathbb{N}$, the study of the functions p_n^* solving

$$\min_{p \in \mathcal{P}_n} \|pf - 1\|_H$$

where $\mathcal{P}_n$ is the set of polynomials of degree less than or equal to n. We will generalize this idea by replacing the function 1 above with an arbitrary function in H. Lastly, we will relate these approximants to (generalized) inner functions when the number of distinct elements of $\{p_n^*\}_{n \geq 0}$ is finite.

Speaker: Dale Frymark (Stockholm University)

Title: Connections Between Boundary Triples and Self-Adjoint Perturbation Theory

Abstract: We set up a rank-two self-adjoint perturbation that represents all possible self-adjoint extensions of a Sturm–Liouville differential operator with limit-circle endpoints. The perturbations are singular in the sense that the perturbing vectors do not belong to the underlying Hilbert space. The set up is mainly constructed from the theory of Boundary Pairs and Boundary Triples, which yield interesting spectral results when compared to those from perturbation theory. The talk will apply these ideas to an example: the classical Jacobi differential operator.

Speaker: Samuel Harris (Texas A&M University)

Title: The Graph Isomorphism Game

Abstract: Given two finite, undirected graphs X and Y (with no loops), one can construct a two-player non-local game, where the object of the game is for the players to convince the referee that they have an isomorphism of the graphs X and Y, in some sense. Depending on the type of entanglement accessible to Alice and Bob, the probability of them winning the graph isomorphism game may change, with one of the more general models being in a commuting operator framework. On the other hand, the graph isomorphism game is an example of what is known as a synchronous game, and associated to it is a certain universal *-algebra defined by the rules of the game. In this talk, we will outline these structures, and we will show the following surprising fact: if the algebra A(X,Y) of the graph isomorphism game for graphs X and Y is non-trivial, then there exists a commuting operator framework strategy that wins the game with probability 1.

Speaker: David Jekel (UCLA)

Title: Non-commutative Transport of Measure for Operator Algebras

Abstract: Given self-adjoint operators $X_1, \ldots, X_d$ and $Y_1, \ldots, Y_d$, it is difficult to tell when the von Neumann algebra generated by the X_j's and Y_j's are isomorphic. Viewing the operators as non-commutative random variables, the isomorphism of von Neumann algebras is equivalent to the existence of a non-commutative function that will push forward the non-commutative probability distribution of $X = (X_1, \ldots, X_d)$ to that of $Y = (Y_1, \ldots, Y_d)$. It was proved by Guionnet, Shlyakhtenko, and Dabrowski that certain nice non-commutative probability distributions known as free Gibbs laws can be transported to the non-commutative Gaussian distribution, and thus the associated von Neumann algebras are all isomorphic. More recently, we have shown that this transport can be done in a lower triangular manner, so that the von Neumann algebra generated by $X_1, \ldots, X_k$ is mapped to the von Neumann algebra generated by $Y_1, \ldots, Y_k$ for $k = 1, \ldots, d$. Furthermore, this transport arises in a natural way as the large-n limit of classical transport of measure for random variables in the space $n \times n$ matrices that approximate $(X_1, \ldots, X_d)$ as $n \to \infty$.

Speaker: Michael Jury (University of Florida=

Title: Factorization in noncommutative function theory: multi-Toeplitz operators and Szegő's theorem

Abstract: A classical theorem of Szegő gives a necessary and sufficient condition for a bounded, nonnegative function on the circle to be the square modulus of (the boundary values of) a bounded analytic function; equivalently this may be stated as a factorization theorem for Toeplitz operators. We consider Popescu's notion of a "multi-Toeplitz" operator, which is a natural generalization of a Toeplitz operator to the setting of (noncommutative)

multivariable operator theory, and prove a version of Szegő's theorem in this setting. (This is joint work with Robert T.W. Martin.)

Speaker: Amudhan Krishnaswamy-Usha (Texas A&M University)

Title: Spectral operators in finite von Neumann algebras

Abstract: Spectral operators are operators which have an associated idempotent valued spectral measure. Dunford showed that these are precisely those operators which are similar to the sum of a normal operator and a quasinilpotent operator, which commute with each other. For operators in a finite von Neumann algebra, Haagerup and Schultz showed there exist certain invariant projections which behave well with respect to the spectral measure. In this setting, we show that if the angles between complementary Haagerup-Schultz subspaces are uniformly bounded away from zero (what we call the Uniformly Non-Zero Angle property), the operator is similar to the sum of a normal operator and a 'sot-quasinilpotent' operator that commute. This lets us characterize spectrality as equivalent to decomposability plus this 'uniformly non-zero angle property'. Using this characterisation, we show that a class of operators from free probability (which include Voiculescu's circular operator) are not spectral. This is joint work with Ken Dykema.

Speaker: Mark Mancuso

Title: Free analysis: a comparison of the matricial and operatorial settings

Abstract: Classically, (free) noncommutative analysis is done over a graded space of tuples of matrices. It has proved fruitful, however, to consider a purely operatorial setting that can be seen as a sort of completion of the classical matrix case. While interesting in its own right, operatorial noncommutative function theory occasionally provides a useful perspective to tackle problems that originate in the matrix setting.

We will provide an overview of this operatorial setting and discuss examples where appealing to operators can give valuable insight into the matrix case. On the other hand, we will also discuss an open problem in the operatorial theory that has been solved in the matrix case.

Speaker: Meredith Sargent (University of Arkansas)

Title: Optimal approximants and orthogonal polynomials in several variables

Abstract: In recent years, optimal polynomial approximants have been used to study cyclicity of functions in Dirichlet-type spaces on the complex unit disk, with particular interest being paid to the connection between the location of the zero sets of the optimal approximants and cyclicity, as well as a correspondence between optimal approximants and orthogonal polynomials. In this talk we discuss generalizing the concept of optimal approximants to several variables, including to the cases of Dirichlet-type spaces on the bidisk,

and to a scale of Drury-Arveson-type spaces on the ball, as well as some of the inherent complications. This is joint work with Alan Sola.

Speaker: Edward Timko (University of Manitoba)

Title: Function Algebras and d-tuples of Model Operators

Abstract: In this talk, we look at algebras generated by d-tuples of model operators associated with certain complete Nevanlinna-Pick spaces. The central question we investigate is when these algebras fail to have completely isometric Gelfand transforms. We connect this with properties of Arveson's noncommutative Choquet boundary, and find a variety of conditions, often on the joint spectrum of the model d-tuple, under which the Gefland transforms of the algebras are not completely isometric. This is joint work with Raphaël Clouâtre.

Speaker: Nicole Tuovila (University of Florida)

Title: Automorphisms of Free Spectrahedra

Abstract: This talk will give two new results about automorphisms of spectraballs. A spectraball is the fully matricial polar dual of an operator space spanned by finitely many matrices. The first result characterizes when a spectraball has a transitive automorphism group at level one. The second one shows that all spectraball automorphisms arise as restrictions of automorphisms of classical matrix balls.

Speaker: Nathan Wagner (Washington University in St. Louis)

Title: Weighted L^p Estimates for the Bergman and Szego Projections on Strongly Pseudoconvex Domains with Near Minimal Smoothness

Abstract: The Bergman and Szego projections are fundamental operators in complex analysis in one and several complex variables. Consequently, the mapping properties of these operators on L^p and other function spaces have been extensively studied. In this talk, we discuss some recent results for these operators on strongly pseudoconvex domains with near minimal smoothness. In particular, weighted L^p estimates are obtained, where the weight belongs to a suitable generalization of the Bekolle-Bonami or Muckenhoupt class. For these domains with less boundary regularity, we use an operator-theoretic technique that goes back to Kerzman and Stein. This technique enables one to express the Bergman or Szego projection as a product of two operators whose mapping properties are more tractable, provided one of the operators is invertible on the space of interest. This talk is based on joint work with Brett Wick.