Abstracts

On Critical Dipoles in Dimensions $n \geq 3$

S. Blake Allan
Baylor University

We reconsider generalizations of Hardy's inequality corresponding to the case of (point) dipole potentials $V_\gamma(x) = \gamma(u,x)|x|^{-3}$, $x \in \mathbb{R}^n \backslash \{0\}$, $\gamma \in [0,\infty)$, $u \in \mathbb{R}^n$, $|u| = 1$, $n \in \mathbb{N}$, $n \geq 3$. More precisely, for $n \geq 3$, we provide an alternative proof of the existence of a critical dipole coupling constant $\gamma_{c,n} > 0$, such that

for all $\gamma \in [0, \gamma_{c,n}]$, and all $u \in \mathbb{R}^n$, $|u| = 1$,
$$\int_{\mathbb{R}^n} d^n x \, |(\nabla f)(x)|^2 \geq \pm\gamma \int_{\mathbb{R}^n} d^n x \, (u,x)|x|^{-3}|f(x)|^2, \quad f \in D^1(\mathbb{R}^n).$$

with $D^1(\mathbb{R}^n)$ denoting the completion of $C_0^\infty(\mathbb{R}^n)$ with respect to the norm induced by the gradient. Here $\gamma_{c,n}$ is sharp, that is, the largest possible such constant, and we discuss a numerical scheme for its computation. Moreover, we discuss upper and lower bounds for $\gamma_{c,n} > 0$.
This is based on joint work with F. Gesztesy.

Minimal Riesz energy on balanced fractal sets

Austin Anderson
Florida State University

We will discuss the asymptotics of best packing and minimal Riesz s-energy on self-similar fractals in the case that the parameter s exceeds the Hausdorff dimension of the fractal. We will primarily focus on recent work completed for the case of a 'balanced' fractal set with dependent contraction ratios. The talk will also address the independent case, highlighting potential differences in the analysis, and pose some open problems in related areas.

Noncommutative Operator Functions and a Strict Nullstellensatz

Meric L. Augat

Washington University in St. Louis

Free Analysis is a burgeoning subfield of analysis that aims to understand noncommutative functions by studying their evaluations on different noncommutative objects.

Recent advances imply that evaluations of noncommutative functions on matrices alone is not always enough to capture the full picture, e.g. positivity of the trace of a symmetric polynomial.

In this talk, we discuss evaluating noncommutative functions on the set of bounded operators on a separable Hilbert space – a natural generalization of evaluating on matrices.

In particular, we will discuss the boundedness of certain noncommutative function balls, a strict noncommutative Nullstellensatz, and an intriguing conjecture (purportedly) relating the algebra generated by a polynomial to its non-injectivity set.

On reducing submodules for a class of Hilbert modules

Shibananda Biswas

IISER Kolkata

Let G be a finite pseudoreflection group, $\Omega \subseteq \mathbb{C}^n$ be a bounded domain which is a G-space and $\mathcal{H} \subseteq \mathcal{O}(\Omega)$ be an analytic Hilbert module possessing a G-invariant reproducing kernel. We study the structure of joint reducing subspaces of the multiplication operator $\mathbf{M}_{\boldsymbol{\theta}}$ on $\mathcal{H}$, where $\{\theta_i\}_{i=1}^n$ is a homogeneous system of parameters associated to G and $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$ is a polynomial map of $\mathbb{C}^n$. This is a joint work with Swarnendu Datta, Gargi Ghosh and Subrata Shyam Roy.

Self-Adjoint Extensions of C-Symmetric Differential Expressions

Michael Bush

University of Delaware

Specifying a domain is a crucial part of defining an operator. In this talk, we will look at two different ways to parameterize the domains on which a class of different expressions are self-adjoint operators. First, through boundary conditions via GKN theory. Then, via finite-rank perturbations with connections to Alexandrov-Clark Theory. We will also examine the connection between these two characterizations of self-adjoint domains.

C*-structure on images of completely positive order zero maps

Kristin Courtney

University of Münster

Completely positive order zero maps are orthogonality preserving maps that also preserve the positive structure of a C*-algebra. Although these maps do not preserve algebraic structure, they are more abundant than *-homomorphisms and actually retain a lot of the same nice properties. A key motivation for the study of these maps is the study of nuclear C*-algebras, which are often portrayed as the C*-analogue to amenable groups or as the C*-algebras that do not admit quantum entanglement.

In this talk, I will report on ongoing work with Wilhelm Winter, where we conduct a thorough study of the images of these maps with the goal of gaining insight into the structure of nuclear C*-algebras.

The Spectral Picture and Joint Spectral Radius of the Generalized Spherical Aluthge Transform

Raúl E. Curto

The University of Iowa

For an arbitrary commuting d–tuple $\boldsymbol{T}$ of Hilbert space operators, we fully determine the spectral picture of the generalized spherical Aluthge transform $\Delta_t(\boldsymbol{T})$ and we prove that the spectral radius of $\boldsymbol{T}$ can be calculated from the norms of the iterates of $\Delta_t(\boldsymbol{T})$.

We first determine the spectral picture of $\Delta_t(\boldsymbol{T})$ in terms of the spectral picture of $\boldsymbol{T}$; in particular, we prove that, for any $0 \le t \le 1$, $\Delta_t(\boldsymbol{T})$ and $\boldsymbol{T}$ have the same Taylor spectrum, the same Taylor essential spectrum, the same Fredholm index, and the same Harte spectrum. We then study the joint spectral radius $r(\boldsymbol{T})$, and prove that $r(\boldsymbol{T}) = \lim_n \left\| \Delta_t^{(n)}(\boldsymbol{T}) \right\|_2$ $(0 < t < 1)$, where $\Delta_t^{(n)}$ denotes the n–th iterate of Δ_t. For $d = t = 1$, we give an example where the above formula fails.

The talk is based on recent research with Chafiq Benhida, Sang Hoon Lee and Jasang Yoon.

Random Interpolating Sequences in the Polydisc and the Unit Ball

Alberto Dayan
Washington University in St. Louis

A random sequences Z in the unit disc is determined by a sequence of deterministic radii and a sequence of i.i.d. random variables uniformly distributed on the unit circle. Chochran and Rudowicz found the 0-1 Kolmogorov law for Z to be interpolating, that is, the cut-off condition on the a-priori fixed radii in order for Z to be interpolating almost surely. In this talk, we will extend their work to random interpolating sequences for bounded analytic functions in the d-dimensional polydisc and for the Besov-Sobolev spaces on the unit ball.

This is a joint work with Brett Wick and Shengkun Wu

Analogues of Finite Blaschke Products

Christopher Felder
Washington University in St. Louis

In a fairly general setting, we will discuss analogues of finite Blaschke products as they relate to inner functions. These functions can be seen as certain linear combinations of reproducing kernels, and as generalizations of a construction given by Shapiro and Shields. Joint work with Trieu Le.

Fourier integral operators in the Borehole geometry

Raluca Felea
Rochester Institute of Technology

In this talk we will describe the microlocal properties of the forward operator F, which maps the image to the data, and which appears in the dense array borehole seismic inverse problems. We will consider two cases: one with no caustics and one with fold type caustics. We will show that in the first case, F is an FIO which satisfies the Bolker condition, and in the second case, F is an FIO which exhibits singularities like submersion with folds and cross caps. To find the image, one considers the normal operator F^*F. In the first case, F^*F is a pseudodifferential operator and in the second case, F^*F is a new type of operator which will produce artifacts. We will describe these artifacts and find their strength.

Reducing subspaces for rank-one perturbations of diagonal operators

Eva A. Gallardo Gutiérrez
Complutense University of Madrid

In this talk we address the existence of reducing subspaces for rank-one perturbations of diagonal operators and, in general, of normal operators of multiplicity one. As we will show, the spectral picture will play a significant role in order to prove the existence of reducing subspaces for rank-one perturbations of diagonal operators whenever they are not normal (Joint work with J. González-Doña)

Quantum Graphs

Priyanga Ganesan
Texas A&M University

Quantum graphs are an operator space generalization of classical graphs. In this talk, I will motivate the idea of a quantum graph and its significance in quantum communication. We will look at the different notions of quantum graphs that arise in operator systems theory, non-commutative topology and quantum information theory. I will then introduce a non-local game with quantum inputs and classical outputs, that generalizes the graph coloring game.

Interpolation in algebras of multipliers on the ball

Michael Hartz
Saarland University

The classical Rudin–Carleson theorem shows that any compact subset E of the circle of linear Lebesgue measure 0 is a peak interpolation set for the disc algebra. This means that given any continuous function h on E, there is a function f in the disc algebra that agrees with h on E and has modulus strictly less than $\|h\|_\infty$ outside of E.

I will talk about generalizations of the Rudin–Carleson theorem and related interpolation theorems to algebras of mutipliers on the ball. In this setting, the appropriate replacements for Lebesgue null sets are called totally null sets. I will explain that in classical Dirichlet type spaces, the totally null sets are precisely the sets of capacity zero. This talk is based on joint works with Ken Davidson and Nikolaos Chalmoukis.

New look on Loewner's theory of matrix monotone functions

Otte Heinävaara

Princeton University

In his seminal 1934 paper, Charles Loewner characterized functions that, when extended via functional calculus, preserve semidefinite order of Hermitian matrices. Surprisingly, such functions are necessarily real analytic, and can be analytically extended to upper half-plane. Loewner's result has since been reproven and extended in various ways. I will discuss a new approach to Loewner's characterization and explain how it can be used to shed a new light to a wide range of related results.

Certain Invariant Spaces of Bounded Measurable Functions on a Sphere

Samuel A. Hokamp

St. Norbert College

In their 1976 paper, Nagel and Rudin characterize the closed unitarily and Moebius invariant spaces of continuous and Lp-functions on a sphere. In this talk, we discuss an analogous characterization for the weak*-closed unitarily and Moebius invariant spaces of L-infinity functions on a sphere. We also investigate the weak*-closed unitarily and Moebius invariant algebras of L-infinity functions on a sphere.

A new proof of a weighted John-Nirenberg Theorem, via sparse operators

Irina Holmes

Texas A&M University

In this talk we revisit a result of Muckenhoupt and Wheeden, which gives a weighted version of the classical John-Nirenberg Theorem (specifically for A_p weights). We will discuss a modern proof of this result, using the recent machinery of sparse operators.

Convergence of hulls of curves

Alexander J. Izzo
Bowling Green State University

The notion of polynomial convexity is fundamental in the study of Banach algebras and polynomial approximation. It is known from work of Forstneric, Rosay, Low, and Wold that generically a totally real, smoothly embedded, compact manifold in C^n having dimension strictly less than n is polynomially convex. For the case of simple closed curves, we establish stronger results regarding genericness of polynomially convex embeddings. We also show that under suitable hypotheses, the polynomial hull of the limit of a sequence of simple closed curves is the limit of the polynomial hulls. This is joint work with Lee Stout.

Non-commutative convolution operations defined by trees (joint work with Weihua Liu & joint work with Ethan Davis and Zhichao Wang)

David Jekel
University of California, San Diego

For rooted subtrees $\mathcal{T}$ of an N-regular tree, we define convolution-type operations $\boxplus_{\mathcal{T}}(\mu_1, \ldots, \mu_N)$, which take N probability measures on $\mathbb{R}$ as inputs and output a single probability measure. These convolution operations form an operad and include many operations that were previously developed in non-commutative probability theory such as free, boolean, and monotone convolution. They are defined by a certain recursive relation that expresses the $\mathcal{T}$-convolution in terms of the convolutions associated to each of the branches of $\mathcal{T}$. We give a limit theorem for $\mathcal{T}$-convolutions analogous to the classical limit theorems about sums of random variables and classical stable distributions. We present pictures and some open problems relating to $\mathcal{T}$-analogs of the classical stable distributions.

The Relationships Between the Henstock Kurzweil and Lebesgue Integrals Constructively Defined on a Separable Banach Space

Timothy Myers
Howard University

Gill and Myers [GM] proved that every separable Banach space, denoted $\mathcal{B}$, has an isomorphic, isometric embedding in $\mathbb{R}^\infty = \mathbb{R} \times \mathbb{R} \times \cdots$. They used this result and a method due to Yamasaki [YA] to construct a sigma-finite Lebesgue measure for $\mathcal{B}$ and defined the associated integral in a way that equals a limit of finite-dimensional Lebesgue integrals.

The objective of this talk is to first define the Henstock-Kurzweil (HK) integral

in the usual way on $\mathcal{B}$, then extend the familiar relationships between the HK and Lebesgue integrals to those on $\mathcal{B}$. Specifically, we will prove that a function which is Lebesgue-integrable on a rectangle $R \subset \mathcal{B}$ is HK-integrable on R to the same value. Conversely, we will show that an absolutely HK-integrable function on R is absolutely Lebesgue integrable on R to the same value.

Our approach is constructive in the sense that the HK integral on $\mathcal{B}$ equals a limit of HK integrals on Euclidean space as the dimension $n \to \infty$.

Keywords : Henstock Kurzweil integral, Lebesgue integral, separable Banach space.

References

[GM] T.L. Gill, T. Myers, *Constructive Analysis on Banach Spaces,* Real Analysis Exchange, **44** (2019) 1-36.

[YA] Y. Yamasaki, *Measures on Infinite Dimensional Spaces,* World Scientific, (1985).

Distance between reproducing kernel Hilbert spaces and geometry of finite sets in the unit ball

Satish Pandey

Technion - Israel Institute of Technology

In this paper we study, in a quantitative manner, the relationships between the structure of a reproducing kernel Hilbert space, the structure of its multiplier algebra, and the geometry of the underlying set. We introduce a variant of the Banach-Mazur distance suited for measuring the distance between reproducing kernel Hilbert spaces, that quantifies how far two spaces are from being isometrically isomorphic as reproducing kernel Hilbert spaces. We introduce an analogous distance for multiplier algebras, that quantifies how far two algebras are from being completely isometrically isomorphic. We show that, in the setting of finite dimensional quotients of the Drury-Arveson space, two spaces are "close" to one another if and only if their multiplier algebras are "close", and that this happens if and only if the underlying point-sets are "almost congruent", meaning that one of the sets is very close to an image of the other under a biholomorphic automorphism of the unit ball. These equivalences are obtained as corollaries of quantitative estimates that we prove. This is a joint work with Danny Ofek and Orr Moshe Shalit.

On the spectrum of Toeplitz operators associated with a constrained subalgebra of H^∞

Douglas Pfeffer
Berry College

Given a symbol $\phi \in L^\infty$, its Toeplitz operator $T_\phi \colon H^2 \to H^2$ is defined by $T_\phi f = P_{H^2}\phi f$, where $P_{H^2} \colon L^2 \to H^2$ is the orthogonal projection. Here, H^∞ is viewed as the multiplier algebra for H^2. Investigations into the spectrum of these Toeplitz operators have been carried out for a variety of symbols ϕ. Recently, similar investigations have taken place for constrained subalgebras of H^∞. In this talk, we focus on the 2-point algebra $\mathscr{A}_{a,b} = \{f \in H^\infty \ : \ f(a) = f(b)\}$ and discuss recent work on determining the structure of the spectrum for its associated Toeplitz operators. This is joint work with Christopher Felder and Benjamin Russo.

One Parameter Semigroups in Two Complex Variables

Michael R. Pilla
Indiana University

For self maps of the disk, it can be shown that under the right conditions one can embed a discrete iteration of the map into a continuous semigroup. In this talk we extend these results to two complex variables for maps of the unit ball into itself under some restricted conditions.

A NOTE ON AN OPEN CONJECTURE IN RATIONAL DYNAMICAL SYSTEM

Zeraoulia Rafik
University of Batna2.Algeria

This note is an attempt with the open conjecture 8 proposed by the authors of [1] which states :

Assume $\alpha, \beta, \lambda \in [0, \infty)$. Then every positive solution of the difference equation:

$$z_{n+1} = \frac{\alpha + z_n\beta + z_{n-1}\lambda}{z_{n-2}}, \quad n = 0, 1, \dots$$

is bounded if and only if $\beta = \lambda$.

We will use a construction of sub-energy function and properties of Todd's difference equation to disprove that conjecture in general.

Keywords:Difference equation; boundedness properties · Todd's equation · sub-energy function **MSC**:39A10; 39A22

References

[1] G. LADAS, G. LUGO AND F. J. PALLADINO OPEN PROBLEMS AND CONJECTURES ON RATIONAL SYSTEMS IN THREE DIMENSIONS, In SARAJEVO JOURNAL OF MATHEMATICS Vol.8 (21) (2012), 311–321

Operators and Hilbert Spaces for Learning in Nonlocal and Higher Order Dynamical Systems

Joel Rosenfeld

University of South Florida

In this talk we investigate generalizations of Liouville operators (often referred to as Koopman generators) that enable us to avoid state augmentation when studying higher order dynamical systems. In data driven methods for dynamical systems, state augmentation requires numerical differentiation, which can produce very noisy data. These operators allow for us to supplant numerical differentiation with numerical integration, which is much more stable.

However, the introduction of these operators requires a new Hilbert space construction that is "signal forward." In particular, whereas with first order systems, the dynamics only depend on the current state. For nonlocal dynamical systems, such as those derived from fractional order integro-differenital equations, a decision requires full knowledge of the history of a state. The operators discussed above for higher order dynamical systems also exhibit this nonlocal behavior. To account for this, we will discuss signal valued reproducing kernel Hilbert spaces.

This is joint work with Rushikesh Kamalapurkar and Benjamin P. Russo.

A Non-Commutative Bayes' Theorem

Benjamin Russo

Farmingdale State College SUNY

In this talk we'll discuss a diagrammatic formulation of Bayes' theorem in quantum Markov categories. We'll discuss a necessary and sufficient condition for the existence of a Bayesian inference in the setting of finite dimensional C*-algebras. In particular, the construction of a Bayesian inverse, when they exist, involves solving a positive semidefinite matrix completion problem for the Choi matrix.

A Multivariable Kaluza's Lemma

Jesse Sautel
University of Tennessee

A lemma due to Theodor Kaluza gives a sufficient condition for a reproducing kernel of one complex variable to be of complete Nevanlinna-Pick type, with the condition stated only in terms of the coefficients of the power series for the kernel. In this talk, I will present a generalized version of this lemma which applies to reproducing kernels of any number of complex variables.

A refinement of Hardy's inequality

Jonathan Stanfill
Baylor University

The principal aim of this talk is to prove the inequality

$$\int_0^\pi dx\, |f'(x)|^2 \geq \frac{1}{4}\int_0^\pi dx\, \frac{|f(x)|^2}{\sin^2(x)} + \frac{1}{4}\int_0^\pi dx\, |f(x)|^2, \quad f \in H_0^1((0,\pi)),$$

where both constants $1/4$ appearing in the above inequality are optimal. In addition, this inequality is strict in the sense that equality holds if and only if $f \equiv 0$. This inequality is derived with the help of the exactly solvable, strongly singular, Dirichlet-type Schrödinger operator associated with the differential expression

$$\tau_s = -\frac{d^2}{dx^2} + \frac{s^2 - (1/4)}{\sin^2(x)}, \quad s \in [0,\infty),\ x \in (0,\pi).$$

The new inequality represents a refinement of Hardy's classical inequality

$$\int_0^\pi dx\, |f'(x)|^2 \geq \frac{1}{4}\int_0^\pi dx\, \frac{|f(x)|^2}{x^2}, \quad f \in H_0^1((0,\pi)),$$

it also improves upon one of its well-known extensions in the form

$$\int_0^\pi dx\, |f'(x)|^2 \geq \frac{1}{4}\int_0^\pi dx\, \frac{|f(x)|^2}{d_{(0,\pi)}(x)^2}, \quad f \in H_0^1((0,\pi)),$$

where $d_{(0,\pi)}(x)$ represents the distance from $x \in (0,\pi)$ to the boundary $\{0,\pi\}$ of $(0,\pi)$.

This is based on joint work with Fritz Gesztesy and Michael Pang.

Weighted properties of compact operators

Cody Bullett Stockdale
Clemson University

The boundedness properties of singular integral operators are of central importance in analysis. Within the last decade, optimal bounds for general Calderón-Zygmund operators acting on weighted Lebesgue spaces in terms of Muckenhoupt weight characteristics have been obtained. In addition to this theory concerning boundedness of Calderón-Zygmund operators, a theory for compactness of these operators has recently been established. The first goal of this talk is to present the extension of compact Calderón-Zygmund theory to weighted spaces using sparse domination techniques. A similar line of research concerns the weighted boundedness of the Bergman projection in terms of Bekollé-Bonami weights, and compactness in this setting is understood within the study of Toeplitz operators. We will also discuss the weighted theory of Toeplitz operators on the Bergman space.

New estimates for dyadic Carleson sequences

Brandon Sweeting
University of Cincinnati

We obtain new sharp estimates for a family of Carleson sequences related to dyadic A_2 weights. The proof uses Bellman functions, but unlike the typical situation found in literature, in our setting these functions do not have infinitesimal extremal splits and do no arise as solutions of a PDE. This is joint work with Leonid Slavin.

Some aspects of positive linear maps and connection with quantum information theory

Wai-Shing Tang
National University of Singapore

Let M_n be the algebra of all n by n complex matrices. We will discuss a decomposition theorem for k-positive linear maps from M_m to M_n, where $2 \leq k < \min\{m, n\}$. Some consequences and connection with quantum information theory will be discussed. This talk is based on joint work, at various stages, with L. Chen, Y. Yang and D. H. Leung.

On the construction of solution for system of partial differential-difference equations

A. Tanuja
Siddaganga Institute of Technology

In this paper, Nevanlinna theory and difference Nevanlinna theory of several complex variables are used to construct meromorphic solution for system of partial differential-difference equation of Fermat type with two complex variables such as

$$(f(z))^m + \left[f(z+c) + \frac{\partial f}{\partial z_1} + \frac{\partial f}{\partial z_2}\right]^m = 1,$$

and

$$\left\{ \begin{array}{l} (f_1(z))^m + \left[f_2(z+c) + \dfrac{\partial f_1}{\partial z_1} + \dfrac{\partial f_1}{\partial z_2}\right]^m = 1, \\ (f_2(z))^m + \left[f_1(z+c) + \dfrac{\partial f_2}{\partial z_1} + \dfrac{\partial f_2}{\partial z_2}\right]^m = 1, \end{array} \right\}$$

which are an extension or generalization of previous theorems given by Xu et al. (2020).

Keywords:
Nevanlinna theory; Complex partial differential-difference equation; Meromorphic solution.

AMS Mathematics Subject Classification 2020:
30D35

Local theory of noncommutative functions

Jurij Volčič
Texas A&M Univesity

This talk addresses local aspects of noncommutative functions. The theory branches in two directions. Noncommutative functions analytic about a scalar point admit a universal skew field of fractions, whose elements are called meromorphic germs. Furthermore, analytic germs about scalar points admit a local-global rank principle. On the other hand, if Y is a non-scalar point, then there exist nilpotent analytic noncommutative functions about Y. Nevertheless, the ring of germs about Y can be described as the completion of the free algebra with respect to the vanishing ideal at Y. This is a consequence of a free Hermite interpolation theorem: if f is a noncommutative function, then for any finite set of irreducible points and a natural number L there exists a noncommutative polynomial that agrees with f at the chosen points up to differentials of order L.

A Békollè-Bonami Class of Weights for Certain Pseudoconvex Domains

Nathan Wagner

Washington University in St. Louis

We prove the weighted L^p regularity of the ordinary Bergman projection on certain pseudoconvex domains where the weight belongs to an appropriate generalization of the Békollè-Bonami class. The main tools used are estimates on the Bergman kernel obtained by McNeal and Békollè's original approach of proving a good-lambda inequality. In particular, we view the Bergman projection as a Calderón Zygmund singular integral operator with respect to a particular quasi-metric. This quasi-metric arises from a scaling approach employed by McNeal and others to obtain estimates on the Bergman kernel for various pseudoconvex domains of finite type. This is joint work with Zhenghui Huo and Brett Wick.

Convergence of Certain Lower Triangular Random Matrices to the Volterra Operator

Tapesh Yadav

University of Florida

We observe some sufficient conditions for convergence of random lower diagonal matrices to the Volterra operator. The mode of convergence is in almost sure sense in SOT and WOT like fashion. We also give results on asymptotic moments for such matrices.

Area operators on Hardy spaces in the unit ball of $\mathbb{C}^n$

Ruhan Zhao

SUNY Brockport

We characterize boundedness and compactness of area operators from H^p in the unit ball of $\mathbb{C}^n$ into $L^q(\mathbb{S}_n)$ in terms of Carleson measures for $0 < p, q < \infty$. This is a joint work with Xiaosong Liu and Zengjian Lou.

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