# Abstracts for Computability Speakers, SEALS 2022 – March 5,6

Calvert wcalvert@siu.edu

**Title: Logic, Analysis, and Learning**

Machine learning, as it is currently used in applications, bears a much greater superficial resemblance to numerical analysis than it does to its roots in 1960's computability theory. However, the field is ripe for a revisiting of these origins. Measurable sets, $\Pi^0_1$ classes, degree theory, and more offer insight into modern approaches to learning, and potentially into the critical barriers to applications.

Cameron Freer freer@mit.edu

**Title: A characterization of properly ergodic structures**

For a countable language *L*, we consider probability measures on the space of *L*-structures with domain the natural numbers that are ergodic and invariant under the logic action. We call any such measure an *ergodic structure*, and say that it is *properly ergodic* when it assigns measure 0 to each isomorphism class of structures. We describe the history of properly ergodic structures in combinatorics and ergodic theory, and present necessary and sufficient conditions for a theory from a countable fragment *F* to be the complete *F*-theory of some properly ergodic structure. We also strengthen a result of Scott and Krauss about properly ergodic structures, and discuss how our methods establish Vaught's Conjecture for sentences having properly ergodic models.

Joint work with Nathanael Ackerman, Alex Kruckman, and Rehana Patel.

Valentina Harizanov harizanv@gwu.edu

**Title: Effective ultraproducts of structures**

The effective ultraproduct construction for structures is a computability-theoretic analog of the classical ultraproduct construction, where the structures are uniformly computable and the role of an ultrafilter is played by a cohesive set. A cohesive set is an infinite set of natural numbers, which cannot be split into two infinite parts by any computably enumerable set. While the elements of the classical ultraproduct are equivalence classes of sequences of elements of structures, in the effective case, these sequences are partial computable, and in some cases computable. Hence the effective ultraproduct construction allows us to build countable non-standard models with interesting properties. For a decidable structure, the effective ultrapower is elementarily equivalent to the structure. However, in general, by Dimitrov's theorem, effective ultrapowers preserve only first-order properties expressed by sentences at lower levels of arithmetical hierarchy. It is possible for isomorphic computable structures to have non-elementarily equivalent effective ultrapowers over a fixed cohesive set. We will present some of our recent collaborative results on effective ultraproducts and ultrapowers for various algebraic structures.

Matthew Harrison-Trainor, matthhar@umich.edu

**Title: Finding an atlas for a closed surface**

A topological manifold is a topological space for which there exists an atlas but, unlike smooth manifolds, the atlas is not part of the structure of the manifold. Given a topological manifold, how hard is it to recover an atlas? We consider this question for closed surfaces and prove that every computable Polish space homeomorphic to a closed surface admits an arithmetic atlas, and indeed an arithmetic triangulation. This is as simple as one could reasonably hope for; essentially, the locally Euclidean structure of a surface can be recovered from the topological structure in a first-order way, i.e., without reference to curves or homeomorphisms or other higher-order objects.

Robert Lubarsky rlubarsk@fau.edu

**Feedback Turing Machines Can Compute the Borel Functions, and Extensions**

Feedback is a computational paradigm by which the oracle contains the con- and divergence information about computations relative to itself. This talk will discuss how they can be used to compute exactly the Borel functions, thereby providing a machine model for Borel computability. It will end with some questions about extending feedback computability by iterating the feedback, and how that might extend the functions computable on the reals.

This is joint work with Nate Ackerman and Cameron Freer.

Neil Lutz lutz@cs.swarthmore.edu

**Title: Dimension and the Structure of Complexity Classes**

The Point-to-Set Principle, which relates the theory of algorithmic information to fractal geometric dimensions, has been a recent avenue for applications of computability theory to geometric measure theory. In this talk, I will describe resource-bounded instances of the Point-to-Set Principle and show how they can be used to explore the fractal structure of complexity classes.

Tim McNicholl, Iowa State mcnichol@iastate.edu

**Title: Presentations and theories of metric structures**

We survey recent work by Camrud, Goldbing Hart, and McNicholl on the complexity of presenting metric structures such as operator algebras and the complexity of their theories.

Justin Miller, postdoc, Dartmouth justin.d.miller@dartmouth.edu

**Title**: **Separating Stochasticity and Randomness**

The law of large numbers says that the average value of random variables tends towards the expected value as the number of trials increases. In computability, the corresponding notion for algorithmically random sets is stochasticity. Random sets are stochastic, but not all stochastic sets are random. We shall separate the computational strength of algorithmically random sets and the injection stochastic sets using the Into and Within set operations. We shall also discussion the application of these techniques to other notions of stochasticity.

Jack Piazza _jap495@psu.edu

**Title: Church stochasticity and uniform** distribution

We present a new framework for constructing notions of randomness based on Church stochasticity. This framework has applications to uniform distribution (UD), a number theoretic concept which has recently been used by Avigad as well as Becher and Grigorieff to develop definitions of randomness. We effectivize classical results about UD, then use the new framework to investigate some properties of these UD randomness notions. Finally, we discuss some potential applications of this framework in computability and Fourier analysis. This is joint work with Jan Reimann.

Christopher Porter, Drake christopher.porter@drake.edu

Title: Continuous randomness via transformations of 2-random sequences

Reimann and Slaman initiated the study of sequences that are Martin-Löf random with respect to a continuous measure, establishing fundamental facts about NCR, the collection of sequences that are not Martin-Löf random with respect to any continuous measure. In the case of sequences that are Martin-Löf random with respect to a computable, continuous measure, the picture is fairly well-understood: such sequences are truth-table equivalent to a Martin-Löf random sequence. However, given a sequence that is Martin-Löf random with respect to a continuous measure but not with respect to any computable measure, we can ask: how effective is the measure with respect to which it is continuously random? In this talk, I will present initial work on this question, discussing several results on sequences that are continuously random with respect to a measure that is computable in 0′.

Diego Rojas, grad student, Iowa State darojas@iastate.edu

**Title: Effective vague convergence of measures on the real line**

Recently, McNicholl and Rojas developed a framework to study the effective theory of weak convergence of measures on the real line. In this talk, we discuss a framework to study the effective theory of vague convergence of measures on the real line. We delineate the distinctions between effective weak and vague convergence, particularly how the distinctions between the classical notions translate to the effective notions in terms of the computability of their limit measures. Moreover, we indicate a sufficient condition for which effective weak and vague convergence of measures coincide.

Don Stull, postdoc Northwestern Donald.stull@northwestern.edu

**Title: Optimal oracles for point-to-set principles**

Recent work has shown a deep connection between algorithmic randomness and (classical) fractal geometry. In particular, there is a growing body of research that uses effective methods to solve problems in fractal geometry which have, on the surface, nothing to do with computability. One of the central theorems of fractal geometry is Marstrand's projection theorem. Let E be an analytic set in the Euclidean plane. For every angle θ, consider the orthogonal projection of E onto the line making angle θ with the origin. Informally, Marstrand's theorem states that, for almost every angle, this projection has maximal Hausdorff dimension. A natural question is whether the analyticity condition can be weakened, or dropped entirely. In this talk, we define the notion of optimal oracles for subsets of Euclidean space. One of the primary motivations of this definition is that, if E has optimal oracles, then the conclusion of Marstrand's projection theorem holds for E. We will show that every analytic set has optimal oracles. We also prove that if the Hausdorff and packing dimensions of E agree, then E has optimal oracles. Thus, the existence of optimal oracles subsume the currently known sufficient conditions for Marstrand's theorem to hold. Under certain assumptions, every set has optimal oracles. However, assuming the axiom of choice and the continuum hypothesis, we construct sets which do not have optimal oracles. This construction naturally leads to a new, algorithmic, proof of Davies theorem on projections.

Francesca Zaffora Blando fzaffora@andrew.cmu.edu

**Title: Algorithmic randomness and Bayesian epistemology**

In this talk, I will provide an overview of some of the applications of algorithmic randomness in Bayesian epistemology. I will focus on two types of results that are part of the basic toolkit of Bayesian epistemologists: convergence-to-the-truth theorems and merging-of-opinions theorems. The former are standardly taken to establish that a Bayesian agent's beliefs are guaranteed to converge to the truth with probability one as the evidence accumulates. The latter are taken to establish that, as long as their initial beliefs are sufficiently compatible, two Bayesian agents are guaranteed to reach inter-subjective agreement with probability one with increasing information. I will focus on two applications of algorithmic randomness in this setting. The first consists in using algorithmic randomness to characterize the collections of data streams on which convergence to the truth occurs, the second in using algorithmic randomness to define notions of compatibility between priors that can be shown to lead to merging of opinions. The results I will discuss about Bayesian convergence to the truth are joint work with Simon Huttegger and Sean Walsh, and they are in line with much recent work in algorithmic randomness aimed at providing characterizations of randomness notions in terms of effectivizations of almost-everywhere convergence theorems in analysis.

Xizhong Zheng, zhengx@arcadia.edu

**Title: On the Definition of Computable Curves**

In mathematics curves are typically defined as the images of continuous real functions (so called *parametrizations*) on a closed interval. They can also be defined as connected one-dimensional compact subsets of points. For simple curves of finite lengths, parametrizations can be further required to be injective or even length-normalized. All of these four approaches to curves are classically equivalent. However, if we define four different versions of *computable curves* based on the effectivization of these four approaches, they are all different,. That is, we get four different classes of "computable curves". More interestingly, these four classes are even *point-separable* in the sense that the sets of points covered by computable curves of different versions are also different. However, if we consider only computable curves of computable lengths, then all four versions of computable curves become equivalent. This shows that the definition of computable curves is robust, at least for those of computable lengths. In addition, we show that the class of computable curves of computable lengths is point-separable from the other four classes of computable curves.

Abstracts for Set Theory Speakers, SEALS 2022 – March 5,6

Anton Bernshteyn

**Title: Borel fractional colorings of Schreier graphs**

Abstract: Fractional coloring is a well-known relaxation of graph coloring. In the Borel context, it was introduced and studied by Meehan. Somewhat surprisingly, we show that for Schreier graphs of Bernoulli shifts of countable groups, the Borel fractional chromatic number always coincides with the obvious lower bound in terms of the measurable independence number. This allows us to asymptotically compute the Borel fractional chromatic number for Schreier graphs of Bernoulli shifts of free groups, answering a question of Meehan.

Udyan Darji

**Title: Local Entropy and Descriptive Complexity**

Abstract:
We investigate local entropy theory, particularly the properties of having uniform positive entropy and completely positive entropy, from a descriptive set-theoretic point of view. We show natural classes of dynamical systems which form Borel sets as well as coanalytic non-Borel sets. In particular, we show that the class of systems with uniform positive entropy and the class of systems with the shadowing property having completely positive entropy is Borel. Meanwhile, the class of mixing systems on a Cantor space is coanalytic but not Borel, and the class of systems on the interval and other orientable manifolds with CPE are coanalytic complete.

Ben Hayes

**Title: Co-spectral radius of equivalence relations**

Abstract: I will discuss joint work with Miklós Abert and Mikolaj Fraczyk. In it, we define the co-spectral radius of inclusions of discrete, probability measure-preserving equivalence relations, which is the exponential decay rate of returning to an S-class. The co-spectral radius is analogous to the spectral radius for random walks on G/H for inclusions of groups. Unlike the group case the almost sure existence of this pointwise limit is nontrivial to establish, and we provide a novel method for proving almost sure existence of such limits. We provide applications of the co-spectral radius to the growth of unimodular random trees, percolation theory, and to Property (T) equivalence relations.

Dakota Ihli

**Title: Topological generators and generic elements in the group of absolutely continuous interval homeomorphisms**

Abstract: We examine the Polish group of absolutely continuous homeomorphisms of the interval, and establish two new results. Firstly, we show that a comeagre set of pairs of elements generate a dense subgroup; secondly, we characterize the generic elements of the group (i.e. elements with comeagre conjugacy class). In this talk I will discuss the second of these results.

Kate Juschenko

**Title: TBA**

Yuqing Frank Lin

**Title: Entropy for actions of free groups under bounded orbit equivalence**

Abstract: Joint work with Lewis Bowen. The f-invariant is a notion of entropy for probability measure preserving (pmp) actions of free groups having some similarities to Kolmogorov-Sinal entropy. Two pmp actions are orbit equivalent if their orbits can be matched almost everywhere in a measurable fashion. Although entropy in general is not an invariant of orbit equivalence, we show that the f-invariant is invariant under the stronger notion of bounded orbit equivalence.

Christian Rosendal

**Title: Amenability, optimal transport and complementation in Banach modules.**

Abstract: Using tools from the theory of transportation cost (aka. Arens– Eells) spaces, three theorems concerning isometric actions of general amenable topological groups on metric spaces with potentially unbounded orbits are established. Specifically, let G be an amenable topological group with no non-trivial homomorphisms to R. Then, when G acts isometrically on a metric space X, the space of Lipschitz functions on the quotient X/G is isometrically isomorphic to a 1-complemented subspace of the Lipschitz functions on X. Similarly, one finds Følner sets with respect to any continuous left-invariant ecart on G and finally every continuous affine isometric action of G on a Banach space has a canonical invariant linear subspace. This generalises previous results of Schneider–Thom and Cuth–Doucha.

Rachel Skipper

**Title: Computing scale in Neretin's group**

Abstract: Neretin's group of spheromorphisms is a natural example of a group with a totally disconnected, locally compact topology which acts on the Cantor set. For an element of the group, we will discuss parts of the process for computing its scale which is related to the dynamics of the element's action on the Cantor set.
This is joint work with Michal Ferov and George Willis.

Riley Thornton

**Title: An algebraic approach to Borel CSPs**

Abstract: This talk will show how algebraic tools for the study of constraint satisfaction problems adapt to the setting of Borel combinatorics. In particular I will show that if a finite constraint satisfaction problem is NP-complete then the corresponding Borel problem is $\Sigma^1_2$-complete (assuming P is not equal to NP). And I will sketch some partial converses.

Felix Weilacher

**Title: Asymptotic separation index and Borel colorings**

Abstract: Asymptotic separation index (asi), introduced recently by Conley et al, is one of several notions from Borel combinatorics measuring the extent to which a Borel graph can be effectively approximated by component-finite subgraphs. We discuss connections between this parameter and Borel coloring problems, along the way proving new upper bounds for edge coloring in a few contexts, as well as lower bounds for vertex coloring. We also discuss the extent to which asi arguments can generalize/unify those from the Baire category and measurable settings.

Lauren Wickman

**Title: Projective Fraïssé Classes to Approximate Knaster Continua and Solenoids**

Abstract: In this talk we will give projective Fraïssé classes that approximate the universal Knaster continuum and the universal solenoid (respectively). We will discuss the automorphism groups of quotients of the limit spaces by their relations, and densely embedding those automorphism groups into subgroups of the homeomorphism groups of their respective continua.

Yuxin Zhou

**Title: Distinguish chromatic numbers for isosceles triangles in choiceless set theory**

Abstract: For n a positive natural number, let Γn be the hypergraph of isosceles triangles on R^n. Under the axiom of choice, the existence of a countable coloring for Γn holds for every n. Without the axiom of choice, the chromatic numbers may or may not be countable. With an inaccessible cardinal, there is a model of ZF+DC in which Γ2 has countable chromatic number while Γ3 has uncountable chromatic number. This result is obtained by a balanced forcing over the symmetric Solovay model.