SEALS 2025 TITLES AND ABSTRACTS

Speaker: Katalin Berlow

Title: Separating complexity classes of problems on grids

Abstract: We study the complexity of labeling problems on Borel graphs induced by free actions of Z^n. Our results separate various complexity classes that were not previously known to be distinct and serve as counterexamples to a number of natural conjectures in the field. In particular, we find a problem which has a measurable solution but no Baire measurable or Borel solution.

Speaker: Jason Block

Title: Computability Theoretic Aspects of Profinite Groups

Abstract: We discuss what it means for infinite profinite groups (all of which are uncountable) to be computable, the complexity of first order theories of these groups, and the degree to which certain countable subgroups form elementary subgroups of profinite groups.

Speaker: Wesley Calvert

Title: Robust Normality

Abstract: Normal numbers were introduced by Borel. Normality is certainly a weak notion of randomness; for instance, there are computable numbers which are absolutely normal. In the present work, we introduce a relativization of normality to a fixed representation system (just as simple normality to base b is normality with respect to that standard representation). When we require normality with respect to large sets of such systems, we find variants of normality that imply randomness notions much stronger than absolute normality.

The primary classes of numbers investigated in this work are the supernormal numbers and the highly normal numbers, which we define. These are relativizations of normality which are, in different senses, robust to all reasonable changes of representation.

Among other results, we give a proof that the highly normal numbers are exactly those of computable dimension 1, which we think gives a more natural characterization than was previously known of this interesting class.

Joint work with Emma Gruner, Elvira Mayordomo, Daniel Turetsky, and Java Darleen Villano.

Speaker: Clinton Conley

Title: Quasi-invariant measures concentrating on countable structures

Abstract: Countable structures whose isomorphism class supports a permutation invariant probability measure have been characterized by Ackerman-Freer-Patel as those with no algebraicity. We provide an analogous model-theoretic characterization of countable structures whose isomorphism class supports a quasi-invariant probability measure. This is joint work with Colin Jahel and Aristotelis Panagiotopoulos.

Speaker: Azul Fatalini

Title: The transcendence degree of the reals over certain set-theoretical subfields

Abstract: It is a well-known result that, after adding one Cohen real, the transcendence degree of the reals over the ground-model reals is continuum. We extend this result for a set X of finitely many Cohen reals, by showing that, in the forcing extension, the transcendence degree of the reals over a combination of the reals in the extension given by each proper subset of X is also maximal. This is joint work with Ralf Schindler.

Speaker: Michel Gaspar

Title: Borel chromatic numbers as cardinal characteristics of the continuum.

Abstract: In this talk we will go through an overview on the recent advances of distinguishing the possible uncountable Borel chromatic numbers for simply defined graphs. In general, this can be achieved in different models of set theory depending on what kind of subgraphs or edges are forbidden to exist (e.g., inexistence of certain cycles, perfect cliques, vertices of uncountable degree etc).

Speaker: Elliot Glazer

Title: Global Choice is needed for classification of topological curves

Abstract: We recently proved that Global Choice is not conservative over ZFC - Foundation. In this talk, we will provide a surprisingly concrete instance of this, showing that the classical topology result that there are exactly four homeomorphism classes of connected 1-dimensional manifolds (not necessarily second countable or set-sized) cannot be proved in ZFC - Foundation, but is provable from this theory plus Global Choice. We will briefly discuss the foundationless multiverse, sketching a proof in ZFC that V is the pure part of some ZFC - Foundation model in which every set is a surjective image of the power class of some definable class long line. In the language of modal logic, we conclude "Global Choice possibly necessarily fails."

Speaker: David Gonzalez

Title: Linear Orderings, Scott Spectra, and New Syntactic Forms

Abstract: The Scott rank of a structure gives an ordinal measure of its descriptive complexity. The analysis of Scott rank has played an important role in countable model theory and computable structure theory for the past several decades. The Scott spectrum of a theory is the set of all possible Scott ranks of the theory. It was suggested by Montalbán in 2013 that the Scott spectrum is well-behaved, in

particular, that any syntactically simple sentence has a model of relatively low Scott rank. It was shown by Harrison-Trainor that this is not true; in fact, there is a sentence of bounded complexity that only has models of any arbitrary Scott rank. In other terms, the "gap" between the complexity of the theory and the complexity of its simplest model can be arbitrarily large.

Since then, more natural examples of theories with a non-trivial gap have been found. Within the last two years, families of example theories extending linear orderings and Peano arithmetic have been found by Gonzalez, Harrison-Trainor, Ho and Montalbán, Rossegger respectively. In recent joint work with Matthew Harrison-Trainor, we produce some contrasting results that instead bound the size of the potential gaps in the case of theories extending linear orderings. This talk will give context for and explore these recent results. We will also talk about some of the syntactic tools used in the proof of our theorems.

Speaker: Emma Gruner

Title: Analyzing Borel Combinatorics in HYP

Abstract: The class of hyperarithmetic sets, or HYP, is one of the weakest set theoretic frameworks which still has sufficient machinery to make sense of Borel sets. In a 2023 paper, Towsner, Weisshaar, and Westrick characterized Borel sets in HYP as exactly those which are Δ_1-definable over L_δ, which in turn coincides with being δ-recursive, where $\delta=\omega_1^{CK}$. This characterization can then be used to analyze classical Borel combinatorics results in the hyperarithmetic setting. In this talk, we will survey some results of this nature from the original paper, as well as some new graph-theoretic results derived from the same techniques.

Speaker: Valentina Harizanov

Title: Ultrahomogeneity and Scott families

Abstract: A computable structure *A* is computably categorical if for every computable isomorphic structure *B* there is a computable isomorphism from *A* to *B*. More generally, *A* is relatively computably categorical if for every isomorphic structure *M* there is an isomorphism from *A* to *M*, which is computable relative to the atomic diagram of *M*. Relative computable categoricity of *A* is equivalent to the existence of computably enumerable Scott family of existential formulas.

Fraisse studied countable structures through the analysis of their age, the set of all finitely generated substructures, up to isomorphism. A structure *A* is called ultrahomogeneous if every isomorphism between finitely generated substructures of *A* extends to an automorphism of *A*. Ultrahomogeneity allows us to construct isomorphisms between *A* and an isomorphic structure using a back-and-forth argument. We will investigate effective categoricity of such structures and their Scott families.

Speaker: Matthew Harrison-Trainor

Title: True stages and descriptive set theory

Abstract: True stage or worker constructions are a key computability-theoretic technique in computable structure theory. They are used to make a computable construction while guessing at some

hyperarithmetic facts. I will talk about how such constructions can be thought of in terms of change-of-topology giving, for example, a proof of Louveau and Saint-Raymond's separation theorem for Borel Wadge classes.

Speaker: Jacob Kowalczyk

Title: Ultraproducts of Finite Sets in ZF + DC

Abstract: It is consistent with ZF + DC that for some ultrafilter *U* on ω, two infinite ultraproducts of finite sets $\prod A_n/U$ and $\prod B_n/U$ have the same cardinality if and only if $0<\lim_U |A_n|/|B_n|<\infty$.In particular, this holds in W[U], where W is the Solovay model and U is $[\omega]^\omega$ -generic.

Speaker: Gabriela Laboska

Title: On the Effectiveness of Partition Regularity over Algebraic Structures

Abstract: Motivated by a recent result by Leader and Russell on inhomogeneous partition regularity that solved a long-standing open question, we approach partition regularity over algebraic structures from computability-theoretic and reverse-mathematical point of view. We specifically analyze a theorem by Straus that has been used either directly or as a motivation to many of the results in this area.

Speaker: Paul Larson

Title: A model of determinacy in which every set of reals is universally Baire

Abstract: The consistency of the theory ZF + AD_R + "every set of reals is universally Baire'' is proved relative to the assumption of ZFC + "there is a cardinal that is a limit of Woodin cardinals and of strong cardinals.'' The proof is based on the derived model construction, which was used by Woodin to show that the theory ZFC + AD_R + ``every set of reals is Suslin'' is consistent relative to ZFC + ``there is a cardinal λ that is a limit of Woodin cardinals and of <λ-strong cardinals.'' The Σ^2_1 reflection property of our model is proved using genericity iterations as in Steel's stationary-tower-free proof of the derived model theorem.

Speaker: Ang Li

Title: Genericity and depth

Abstract: An infinite binary sequence is Bennett deep if the difference between time-bounded prefix-free Kolmogorov complexity and prefix-free Kolmogorov complexity of its initial segments is eventually unbounded for any computable time bound. It is known that every weakly 2-generic set is not deep and there are deep sets of 1-generic Turing degrees. In this talk, we show that there is a deep 1-generic set.

Speaker: Patrick Lutz

Title: An analytic equivalence relation with an unexpected property

Abstract: In his thesis, Tyler Arant introduced and studied the concept of Borel graphability of analytic equivalence relations. Briefly stated, an analytic equivalence relation E on X is Borel graphable if there is a Borel graph on X whose connectivity relation coincides with E. Recently, Tyler Arant, Alexander Kechris and I discovered a surprising phenomenon related to Borel graphability: there is an analytic equivalence relation whose Borel graphability is equivalent to the existence of a non-constructible real. Moreover, this equivalence relation is not an artificial example explicitly designed to have this behavior, but rather one which is familiar to any computability theorist. I will discuss the context for this result and the main ideas used in the proof.

Speaker: Keng Meng Ng, Nanyang Technological University (NTU), Singapore

Title: The algorithmic aspects of continuous mathematics

Abstract: We discuss and survey some recent results in effective Polish spaces and topological spaces. We describe how algorithms play a part in understanding and calibrating the effective content of some continuous spaces and processes, allowing us to view these objects in a different light. Many fundamental questions have not been fully explored until very recently and we will describe some directions in clarifying the basic notions of presentation, duality and isomorphism.

Speaker: Spyridon Petrakos

Title: McDuff factors from dynamical alternating groups

Abstract: Property McDuff is a regularity condition that was first used in order to distinguish group von Neumann algebras of ICC groups, and later played a crucial role in Connes' celebrated classification of injective factors. It also conceptually inspired similar properties appearing in the classification of C*-algebras and non-commutative dynamics, as well as measurable dynamics and equivalence relations.

After briefly introducing all the necessary material from the theory of von Neumann algebras, we will show the existence of simple finitely generated non-amenable groups which give rise to McDuff factors. Time permitting, we will discuss the application of analogous proof techniques in different contexts of a similar flavour.

Speaker: Antoine Poulin

Title: Borel complexity of the isomorphism of Archimedean orderable groups

Abstract: In 2020, Calderoni, Marker, Motto-Ros and Shani asked what the Borel complexity of the isomorphism relation of Archimedean orders on $\mathbb{Q}^n$ is. We answer this by showing that for n greater than 3, it is not hyperfinite and for n greater than 4, it is not treeable. In this talk, we sketch out the arguments, which rests upon Zimmer's notion of amenability and rigidity results due to Popa and Vaes.

Speaker: Luke Serafin

Title: Concerning Prelinearization of Analytic Preorders and Economics

Abstract: A prelinearization of a preorder is an extension to a prelinear order which preserves strict inequalities of the preorder. This notion generalizes the concept of linearizing a partial order, and arises naturally in the economic theory of social choice via what are called social welfare orders. It is known that prelinearizations of analytic preorders do not generally exist in the Solovay model, and we study the

amount of choice needed to construct them. Specifically we want to know whether the existence of prelinearizations of particular analytic preorders implies the existence of nonprincipal ultrafilters on omega. This question was asked in the special case of two types of social welfare orders by Dubey and Laguzzi. We construct, for a given analytic preorder with the property that an element of the ground model may be interpolated between mutually generic elements of the underlying space, a model of ZF+DC in which the preorder admits a prelinearization and which contains no nonprincipal ultrafilter on omega. This is accomplished using the geometric set theory notion of (3,2)-balanced forcing, and in particular solves both problems of Dubey and Laguzzi about social welfare orders.

Speaker: Assaf Shani

Title: Generic dichotomy for homomorphisms for E_3

Abstract: We prove a structural dichotomy for Borel homomorphisms from E_3 to any other analytic equivalence relation. As a corollary, we prove that E_3 is a prime equivalence relation, answering a question of Clemens.

Speaker: Donald Stull

Title: Optimal oracles and the point-to-set principle

Abstract: The point-to-set principle has been used to prove several results in classical geometric measure theory. For example, it has been used to generalize Marstrand's projection theorem, a fundamental result in GMT. In this talk, we introduce an extension of the point-to-set principle - the notion of optimal oracles for subsets of Euclidean space. One of the primary motivations of this definition is that the existence of optimal oracles is the weakest known sufficient condition for Marstrand's projection theorem to hold. In this talk we will discuss sufficient conditions for a set to have optimal oracles. We will also discuss further uses of optimal oracles in geometric measure theory, beyond projection theory.

Speaker: Java Darleen Villano

Title: Computable categoricity relative to a degree

Abstract: A computable structure A is computably categorical relative to a degree d if and only if for all d-computable copies B of A, there is a d-computable isomorphism between A and B. In this talk, we discuss how this relativization behaves chaotically in the c.e. degrees, how it behaves in the generic degrees, and observe for which classes of structures can an example exist which witnesses the chaotic behavior mentioned above.

Speaker: Michael Wolman

Title: Borel uniformization over quotients

Abstract: Given Borel equivalence relations E, F on Polish spaces X, Y and an ExF-invariant Borel subset P of XxY, an invariant uniformization of P is an ExF-invariant subset Q of XxY whose sections contain exactly one F-class. We discuss the (non)-existence of such uniformizations in various contexts, such as when F is smooth or when the sections of P contain countably-many F-classes. Part of this presentation is based on joint work with Alexander Kechris.