SEALS 2024 TITLES AND ABSTRACTS

Speaker: Aaron Anderson

Title: Distality in Continuous Logic

Abstract: We examine distal theories and structures in the context of continuous logic, providing several equivalent definitions. By studying the combinatorics of fuzzy VC-classes, we find continuous versions of (strong) honest definitions and distal cell decompositions. By studying generically stable Keisler measures in continuous logic, we apply the theory of continuous distality to analytic versions of graph regularity. We will also present some examples of distal metric structures, including dual linear continua and a continuous version of o-minimality.

Speaker: Vitaly Bergelson

Title: Hindman's Finite Sums theorem and Ergodic Ramsey Theory

Abstract: A set S of positive integers is called an IP set if there exists an infinite sequence $x_1,x_2,x_3,... \in \mathbb{N}$ such that S consists of the numbers x_i together with all finite sums $x_{i1} + x_{i2} +...+x_{ir}$ with $i_1 < i_2 < ... < i_r$, $r \in \mathbb{N}$. Hindman's finite sums theorem, one of the central results in Ramsey theory, states that for any finite partition of the natural numbers, there is a cell of the partition which contains an IP set.

While the traditional ergodic theory concerns itself with the study of the limiting behavior of various Cesàro averages, the IP ergodic theory utilizes the notion of IP convergence which is based on Hindman's finite sums theorem. This allows one to refine and enhance the classical results and obtain, via the Furstenberg correspondence principle, strong applications to additive number theory and combinatorics. Examples of such applications include various multidimensional and polynomial extensions and refinements of the celebrated Szemerédi's theorem which asserts that any subset of the natural numbers with positive upper density contains arbitrarily long arithmetic progressions.

We shall discuss the connections between Hindman's theorem and the topological algebra in the Stone–Čech compactification of $\mathbb{N}$, survey some of the applications of the IP ergodic theory to combinatorics and formulate some interesting problems and conjectures.

Speaker: Will Brian

Title: Does P(ω)/fin know its right hand from its left?

Abstract: I'm going to talk about an old question of van Douwen: *Are the shift map and its inverse conjugate in the automorphism group of* P(ω)/fin*?* By the late 1980's, van Douwen and Shelah proved that it is consistent they are not conjugate. Specifically, any automorphism witnessing their conjugacy would need to be nontrivial (van Douwen), but it is consistent that all automorphisms are trivial (Shelah). In this talk I'm going to sketch a proof of the complementary result: it is consistent that the shift map and its inverse are conjugate and, in fact, it follows from CH.

Speaker: Ronnie Chen

Title: Étale bundles of countable structures and separable metric structures

Abstract: An étale structure is a "continuous bundle" of first-order structures parametrized over a topological space. We give an introduction to this concept from sheaf theory, and show how it serves as a natural context for generalizations of many tools from countable model theory, such as Polish spaces of countable structures, the Lopez-Escobar theorem, the omitting types theorem, and Scott ranks. If time permits, we will also discuss ongoing work on extending these ideas to continuous logic.

Speaker: Artem Chernikov

Title: Intersecting sets in probability spaces and Shelah's classification

Abstract: For any fixed n and e > 0, given a sufficiently long sequence of events in a probability space all of measure at least e, some n of them will have a common intersection. This follows from the inclusion-exclusion principle. A more subtle pattern: for any $0 < p < q < 1$, we can't find events A_i and B_i so that the measure of A_i intersected B_j is less that p and of A_j intersected B_i is greater than q for all $1 < i < j < n$, assuming n is sufficiently large. This is closely connected to a fundamental model-theoretic property of probability algebras called stability. We will discuss these and more complicated patterns that arise when our events are indexed by multiple indices. In particular, how such results are connected to higher arity generalizations of de Finetti's theorem in probability, structural Ramsey theory, hypergraph regularity in combinatorics, and model theory (no prior knowledge is expected - all of these will be introduced).

Speaker: Gabriel Conant

Title: An analytic version of stable arithmetic regularity

Abstract: In 2011, Malliaris and Shelah proved a strong form of Szemeredi's regularity lemma for the class of "stable graphs", which are graphs omitting a certain special subgraph called a ``half-graph". A group theoretic analogue of their result for finite abelian groups was later obtained by Terry and Wolf using Fourier analytic methods from additive combinatorics. A suitable generalization to arbitrary finite groups was then proved by myself, Pillay, and Terry using model theoretic methods. This talk will focus on an analytic analogue of stability defined for functions, rather than graphs. Roughly speaking, the main result of the talk says that if G is amenable, then any stable function on G is almost constant on all translates of a unitary Bohr set in G of bounded complexity. The proof of this result uses ingredients from topological dynamics and continuous model theory. I will also explain how this result leads to a very short proof of Bogolyubov's Lemma (a variant of the Breuillard-Green-Tao Theorem in finite groups) for arbitrary amenable groups. This is joint work with Anand Pillay.

Speaker: Natasha Dobrinen

Title: Coloring pseudotrees

Abstract: This is joint work with David Chodounský, Monroe Eskew, and Thilo Weinert. We obtain upper bounds for the big Ramsey degrees of chains of size two in the 2-branching pseudotree. Previous work showed that antichains of size two in the pseudotree do not have finite big Ramsey degrees. We develop topological Ramsey spaces for copies of a fixed pseudotree. These Ramsey spaces are used to prove upper bounds for chains, while providing additional clarity for the case of antichains. While the members of the Ramsey space are coding trees with infinitely many unary relations, the proof of the pigeonhole uses no forcing argument, but only induction arguments with ω+1 many judiciously chosen applications of Halpern-Läuchli.

Speaker: Sohail Farhangi

Title: Van der Corput's difference theore, The Ergodic Hierarchy of Mixing, and noncommutative ergodic theorems.

Abstract: We will begin by reviewing the formulation of van der Corput's difference theorem (vdCdt) for uniformly distributed sequences as well as Bergelson's reformulation for sequences of vectors in a Hilbert space. We will then review known results in ergodic theory that were proven using the Hilbertian restatement of vdCdt, as well as the ergodic hierarchy of mixing properties of a unitary operator. One of our main theorems will be establishing a connection between variations of vdCdt and the ergodic hierarchy of mixing in order to obtain generalizations of vdCdt. We will conclude by applying the newly found generalizations of vdCdt to obtain new ergodic theorems for certain classes of noncommuting operators.

Speaker: Daniel Glasscock (University of Massachusetts Lowell)

Title: On sets of pointwise topological recurrence

Abstract: Recurrence of points and sets is a central topic in dynamics that has, for the last half century, found numerous applications in combinatorics and additive number theory. “Sets of topological recurrence” – integer times at which any open set is guaranteed to return to itself under a continuous map of a compact metric space – are well studied. In this talk, we introduce the class of “sets of pointwise topological recurrence,” integer times at which any point is guaranteed to return to a neighborhood of itself. We will show that this class is markedly different from its set analogue, and we will explain some of the anticipated combinatorial and number-theoretic connections. This talk is based on ongoing joint work with Anh Le (University of Denver).

Speaker: John Griesmer

Title: Ultraproduct correspondence principles for sumsets

Abstract: Nonstandard analysis has been applied to additive combinatorics since the seminal work of Renling Jin on the sumset phenomenon. This talk will survey some of the inverse theorems and direct theorems on sumsets proved using ultraproducts and Loeb measure, highlighting opportunities for additional development of the theory.

Speaker: Vince Guingona

Title: Products of classes of structures.

Abstract: Given two classes of structures K_0 and K_1, there are several ways to define the ``product'' of K_0 and K_1, including the direct (or full) product, the lexicographic product, and the free superposition. We will discuss these various products and examine which properties of classes are preserved under each of them.

We also examine the relationship between these products and ``configurations'': Given a class of structures K, let C(K) denote the family of all complete, first-order theories with infinite models that admit a configuration indexed by K. Then, for each of the three products mentioned above, $C(K_0 \cdot K_1)=C(K_0 \cap K_1)$.

The work presented is joint with M. Parnes and L. Scow}.

Speaker: James Hanson

Title: Independence in arbitrary theories via automorphism groups and large cardinals

Abstract: Morley sequences in simple first-order theories can be characterized in terms of Lascar automorphism groups: *I* is a Morley sequence over *A* if and only if for any *J+K* realizing the same EM-type as *I* over *A*, *Autf(U/A)* is generated by the union of *Autf(U/AJ)* and *Autf(U/AK)*. In arbitrary theories, this can be taken to be the definition of a kind of generic indiscernible sequence, generalizing non-dividing Morley sequences and tree Morley sequences (in $NSOP_1$theories). These sequences are universal witnesses of Lascar strong type in a particularly strong way. In the presence of sufficiently large Erdős cardinals, we are able to use indiscernible tree technology to prove the existence of such sequences in arbitrary theories over arbitrary sets of parameters. Some weaker statements are also provable unconditionally.

Speaker: Sumun Iyer

Title: Extremely amenable groups of homeomorphisms

Abstract: A topological group is extremely amenable if every continuous action of it on a compact Hausdorff space has a fixed point. We discuss a construction due to Uspenskij which gives a condition equivalent to extreme amenability for the setting of homeomorphism groups of compact metrizable spaces. We then show a Ramsey-type statement for subsets of simplices that, together with Uspenskij's construction, gives a new proof of a theorem due to Pestov: that the group of orientation-preserving homeomorphisms of the closed unit interval is extremely amenable. This is a joint work with Lukas Michel and Alex Scott.

Speaker: Miriam Parnes

Title: Indivisibility for Classes of Graphs

Abstract: Indivisibility is a property of classes of structures which is related to the Ramsey Property. One motivation for studying indivisibility is that configurations are easier to understand when the index class is indivisible.

In this talk, we will discuss the results of the 2023 Towson University REU on indivisibility of classes of finite graphs. We will first consider indivisibility for hereditarily sparse graphs. Then we will turn our attention to a number of classes of graphs which are characterized by forbidden induced subgraphs, including cographs and perfect graphs. This is joint work with Vince Guingona, Felix Nusbaum, Zain Padamsee, Christian Pippin, and Ava Zinman.

Speaker: Jennifer Pi

Title: An Absence of Quantifier Reduction for II_1 Factors, using Quantum Expanders

Abstract: In this talk, we discuss work on the question of quantifier reduction for tracial von Neumann algebras. (Tracial von Neumann algebras are "nice" classes of bounded operators on Hilbert spaces, and include matrix algebras.)

We prove a complete classification for which tracial von Neumann algebras admit complete elimination of quantifiers, and introduce concrete tests for quantifier elimination. Furthermore, we show that no II_1 factor satisfying a mild ultrapower embedding assumption has a theory that is model complete by using Hastings' quantum expanders. This is joint work with Ilijas Farah and David Jekel.

Speaker: Anand Pillay

Title: Approximate subgroups and topological dynamics.

Abstract: (Joint with Krzysztof Krupinski.) A (k-)approximate subgroup of of an arbitrary group is a subset X of the group which is symmetric and such that the set XX of products of X with itself is contained in the union of finitely many (k) left translates of X by elements of the subgroup <X> generated by X. In a 2020 preprint (Beyond the Lascar group) Hrushovski gave a description of approximate subgroups in terms of quasi-homomorphisms to locally compact groups. The methods involve new and somewhat difficult notions; definability patterns, local logics,...

We give accounts of the results using instead topological dynamics machinery, more precisely using a generalization of topological dynamics to the actions of suitable groups on locally compact (rather than compact) spaces. (If there is time, I will mention connections to approximate lattices of locally compact groups.)

Speaker: Alejandro Poveda

Title: The gluing property

Abstract: In this talk, I'll report on a recent joint work with Y. Hayut where we introduce a new compactness principle called ''The Gluing Property''. This concept was isolated from a former argument by Gitik saying that the existence of a κ-compact cardinal entails an inner model with a strong cardinal. During the talk we shall introduce the gluing property and show that compact-like cardinals (such as strong compacts, Π^1_1-subcompacts, etc) do satisfy a certain amount of gluing. Contrarily, non-compact cardinals like strong cardinals are known to fail to have the gluing property. We shall conclude the exposition by discussing how to force the ω-gluing property from optimal assumptions. This latter being surprisingly mild - just the existence of a measurable cardinal with Mitchell's order ω_1.

Speaker: Nicholas Ramsey

Title: Some model theory of quadratic forms

Abstract: Vector spaces over finite fields with quadratic forms are an important example in the Cherlin-Hrushovski theory of Lie coordinatization of smoothly approximable theories, which are homogeneous structures that can be approximated by finite homogeneous substructures. These come in two forms: the orthogonal spaces (which have only one quadratic form) and the quadratic geometries (which have a whole family of quadratic forms). In work with Charlotte Kestner, we address several basic questions about the model theory of these structures where the field is allowed to be infinite. For example, we classify all pseudo-finite theories of orthogonal spaces and quadratic geometries, axiomatize the model companions of each, and give a reasonably complete neostability-theoretic classification of all of these theories.

Speaker: Atticus Stonestrom

Title: F-generic types in NIP groups

Abstract: In stable group theory, a key role is played by "generic" types. In the more general setting of definably amenable NIP groups, these no longer suffice, and the appropriate substitute is given by "f-generic" types. I will first discuss some of the theory of definably amenable NIP groups, and then present a technical result: for an NIP group, definable amenability is equivalent to the existence of f-generic types. This answers a question of Chernikov and Simon.

Speaker: Le Thai Hoang

Title: Bohr sets in sumsets

Abstract: A Bohr set in an abelian group G is a subset of the form B(K, ε) = {g in G: for all χ in K, |χ(g) - 1| < ε holds}, where K is a finite subset of the dual group of G. A classical theorem of Bogolyubov says that if A is a set of integers of positive upper density δ, then A+A-A-A contains a Bohr set B(K, ε) where |K| and ε depend only on δ. While the same statement for A-A is not true (a result of Kříž), Bergelson and Ruzsa proved that if r+s+t=0, then rA + sA+tA contains a Bohr set (here rA = {ra: a in A}). I will discuss this phenomenon in compact abelian groups and countable discrete abelian groups, as well as analogous results for partitions. This talk is based on joint works with Anh Le and John Griesmer.

Speaker: Rigoberto Zelada

Title: Odd polynomials, Diophantine approximations and applications to ergodic theory.

Abstract: Let $v(x)=a_1x+...+a_Nx^{2N-1}$ be an odd real polynomial. We will start by describing new Diophantine results pertaining to sets of the form {n\in N: | ||v(n)||<ε}, where ||·|| denotes the distance to the closest integer. The second part of the talk will be devoted to applications of these Diophantine results (and the techniques behind them) to ergodic theory and combinatorics. Among other things, we will discuss a variant of the polynomial Szemerédi theorem. The talk is based on joint work with Dr. Bergelson.

Speaker: Andy Zucker

Title: Big Ramsey degrees and topological dynamics

Abstract: We introduce big Ramsey degrees, big Ramsey structures, and completion flows. For Fraisse classes which admit a big Ramsey structure, the automorphism group of the Fraisse limit admits a universal completion flow which is furthermore metrizable and unique up to isomorphism. We then speculate about possible generalizations to other topological groups. Namely, are there dynamical objects which always exist for any topological group, are “suitably unique,” and for automorphism groups of Fraisse structures, are metrizable exactly when the Fraisse class has finite big Ramsey degrees?