SEALS23 ABSTRACTS

Speaker: Matt Bowen

Title: Monochromatic products and sums in $\boldsymbol{N}$ and $\boldsymbol{Q}$

Abstract: We show that any 2-coloring of $\boldsymbol{N}$ and any finite coloring of $\boldsymbol{Q}$ contains a monochromatic set of the form *{x,y,x+y,xy}*, as well as several generalizations. Our techniques rely on some new structural tools for colorings that are related to minimal left ideals in $\beta\boldsymbol{N}$. This talk is partially based on joint work with Marcin Sabok.

Speaker: Joseph Briggs

Title: Infinitary Drisko

Abstract: An elegant result in design theory and extremal combinatorics states that any filling of an $n \times 2n-1$ array with distinct symbols in each row has a full transversal, namely a collection of n cells from distinct rows and columns each with all entries distinct. The famous Ryder-Brualdi-Stein Conjecture from the 70's suggests that for transversals of size $n-1$, such an $n \times n$ array suffices. This mysterious jump from n to $2n-1$ rows remains elusive, so we embark on a discussion of infinite variants in the hope of shedding some light on the underlying combinatorics.

Speaker: Filippo Calderoni

Title: A descriptive view of left-orderable groups

Abstract: In this talk we will discuss some results on left-orderable groups and their interplay with descriptive set theory. We will see how Borel classification can be used to analyze the conjugacy action of a countable left-orderable group *G* on its space of left-orderings LO(*G*). Building on previous work of Calderoni and Clay [1], we will discuss new algebraic conditions for on-smoothness of the conjugacy relation on LO(*G*). Time permitting, we will discuss how our techniques apply to some fundamental groups of 3-manifolds. This is joint work with Adam Clay.

[1] Filippo Calderoni and Adam Clay. Borel structures on the space of left-orderings. Bull. London Math. Soc., 54 (2022), no. 1, 83 -- 94.

Speaker: Wesley Calvert

Title: Structural Highness Notions

Abstract: A Turing degree is said to be low for isomorphisms if it only computes isomorphisms between structures that are computably isomorphic. By contrast, a degree is said to be *high for isomorphism* if it computes an isomorphism of any isomorphic pair of computable structures. To understand this class of (very powerful) degrees, we consider several other structurally motivated classes of degrees, with such related issues as Scott analysis, jump hierarchies, and computing descending sequences in linear orderings. Joint work with Johanna N. Y. Franklin and D. Turetsky.

Speaker: Wesley Calvert (Friday colloquium)

Title: Random Algebraic Extensions of ***Q***.

Abstract: What does a "typical" field look like? To answer this question, we explore a notion of algorithmic randomness for fields, making use of the Haar measure on the absolute Galois group. We prove the existence of algebraic extensions of ***Q*** which are random in this sense, and that this notion does not coincide with the set of noncomputable algebraic fields. Joint work with Valentina Harizanov and Alexandra Shlapentokh.

Speaker: Alex Creiner

Title: On the Ramsey Largeness of Sets of Oracles Separating Complexity Classes

Abstract: The Random Oracle Hypothesis, originally posed by Bennett and Gill in 1981, conjectured that if two complexity classes related to each other in a certain way 'with probability one' then they must actually be related that way. This was quickly disproven, most famously by Chang in 1994 who demonstrated that IP is properly contained in PSPACE with probability one despite the classes previously being shown to equal. In my talk, I will propose what might be considered a strengthening of this hypothesis, utilizing a less conventional notion of largeness than what is implicit to the original (Lebesgue measure 1). Specifically, I define large to mean comeagre in the Ellentuck topology. I will show that in this new context, the set of oracles separating P and NP is not small, and that furthermore showing largeness would be sufficient for P to not equal NP in reality. With the remaining time I will outline similar results concerning the comparison between PH and PSPACE, as well as that between NP and BQP. If time permits I will also discuss the descriptive complexity of all three of these oracle sets.

This is a joint work with Stephen Jackson.

Speaker: Damir Dzafarov

Title: Strong computable reducibility and the tree labeling method

Abstract: In recent years, there has been intense interest in computable combinatorics and computable analysis in various kinds of reducibilities between computational problems. These are problems of the form, "For every real X with some property, there is a real Y with some other property". The most widely studied reducibilities here are computable reducibility and Weihrauch reducibility. One variant of these, strong computable reducibility, behaves somewhat differently from the other two, and surprisingly, turns out to have a deeply set-theoretic character. I will survey this reducibility, and illustrate it with some combinatorial examples. I will then introduce the so-called tree labeling method, introduced in Dzhafarov (2016) and Dzhafarov, Patey, Solomon, and Westrick (2017), which has proved to be a very useful tool for separating problems under strong computable reducibility. I will also discuss some of its set-theoretic features, and some more recent applications of it by David Nichols (2019) and Noah Hughes (2021). The talk will conclude with some open problems.

Speaker: Jan Grebík

Title: Borel graphs with large Borel chromatic number

Abstract: Given a local graph coloring problem, it is natural to ask when we can solve it in a measurable way (in various senses) on a given locally finite Borel graph. A basic example of such a local problem is the proper vertex coloring problem. In a groundbreaking result, Marks gave an example of *d*-regular acyclic Borel graph that does not admit a Borel vertex coloring with *d* colors. In this talk I will discuss new examples of this phenomena, i.e., I will describe how to construct bounded degree acyclic Borel graphs with large Borel chromatic number. The main idea, rather surprisingly, comes from the adaptation of Marks's technique to the LOCAL model of distributed computing. This is a joint work with Brandt, Chang, Grunau, Rozhoň and Vidnyánszky, and with Vidnyánszky.

Speaker: Jan Grebík (Monday colloquium)

Title: Descriptive set theory and distributed computing

Abstract: The fact that a given graph has chromatic number *k* in ***N*** does not necessarily mean that there is an efficient way to produce a *k*-coloring.

This phenomenon has been studied from different perspectives in various areas of mathematics and computer science.

I will discuss recent results that formally connect a part of descriptive set theory that studies the existence of measurable solutions to local coloring problems on Borel graphs, so-called Descriptive graph combinatorics, and the LOCAL model of distributed computing that studies the existence of efficient distributed algorithms that produce solutions to local coloring problems on finite graphs.

Speaker: Rachel Greenfeld

Title: Tiling, periodicity, and decidability

Abstract: Translational tiling is a covering of a space (e.g., Euclidean space) using translated copies of a building block, called a "tile'', without any positive measure overlaps. What are the possible ways that a space can be tiled? Can we decide whether a given set is a tile? It turns out that these questions are closely related.

One of the most well-known conjectures in this area is the periodic tiling conjecture. It asserts that any tile of Euclidean space can tile the space periodically. In a joint work with Terence Tao, we construct a counterexample to this conjecture. In the talk, I will survey the study of the decidability and the periodicity of tilings and discuss our recent progress.

Speaker: Valentina Harizanov

Title: Orders, topology, and computability

Abstract: Orders on algebraic structures are ubiquitous in mathematics and have been studied since Dedekind. For an algebraic structure with a binary operation we consider total orderings of the elements of the domain, which are left-invariant (right-invariant, or both) with respect to the operation. Important examples of such structures include those with associative operations, but also those with not necessarily associative but self-distributive operations. For an orderable structure there is a topology on the set of all left (right, bi-) orders. One of our goals is to connect topological properties with computability-theoretic ones for orders on computable structures. In some cases, spaces of orders on countable structures are homeomorphic to the Cantor set or contain a copy of the Cantor set, and these structures may or may not have orders in all Turing degrees. The question whether the space of bi-orders of a free group of finite rank greater that one is homeomorphic to the Cantor set still remains open.

Speaker: Denis Hirschfeldt

Title: Mycielski's Theorem for Measure in Computability Theory and Reverse Mathematics

Abstract: Let $(A)^n$ denote the set of ordered *n*-tuples of elements of *A*. Mycielski showed that for any sequence of relations R_k for natural *k* with $R_k \subset [0,1]^{\{n_k\}}$ such that each R_k has measure one, there is a homeomorphic copy $C \subset [0,1]$ of the Cantor space such that $(C)^{\{n_k\}} \subset R_k$ for all *k*. As noted by Miller and Yu, this theorem can be seen as the relativized version of the statement that there is a perfect tree *T* such that any join of finitely many paths is random in the sense of Martin-Lof. This fact allows us to give a computability-theoretic and reverse-mathematical analysis of Mycielski's Theorem. In joint work with Carl Jockusch and Paul Schupp, we showed that such a *T* can be built 0'-computably, and used that construction to show that this version of Mycielski's Theorem is provable in ACA_0. On the other hand, we also showed that it is not provable in $WWKL_0$ (which it is easily seen to imply). Whether it is provable in WKL_0, and whether it implies WKL_0 or ACA_0, remain open questions.

Speaker: Stephen Jackson

Title: Some applications of forcing and hyperaperiodicity

Abstract: We present some recent results showing how a combination of forcing and hyperaperiodicity arguments can be used to obtain various results about Borel marker structures. These include various questions about Borel lining, treeing, and marker structures.

Speaker: Liling Ko

Title: A computably small set that is not intrinsically small

Abstract: We construct a set *A* that is computably small but not intrinsically small. To understand these terms, we liken *A* to a game show host playing against a class of computable contestants, analogous to an infinite variant of the Monty Hall problem. The host has infinitely many doors arranged in a line, and each door hides either a goat or a car. A contestant selects infinitely many doors to open and wins if a non-zero density of the selected doors contain a car. Contestants that are disorderly can select doors out of order, opening door i after door $j>i$. Are disorderly contestants more difficult to beat than orderly ones? This is known to be true if contestants are allowed to be adaptive, where they may choose a different door depending on the outcomes of the previously opened ones [1] (via the theorem that MWC-stochasticity 0 does not imply Kolmogorov-Loveland-stochasticity 0). We give a constructive proof to show that the statement also holds in the non-adaptive setting. The construction sheds light on what it takes to outperform order. This is joint work with Justin Miller.

[1] Merkle, Wolfgang and Miller, Joseph S and Nies, Andre and Reimann, Jan and Stephan, Frank, Kolmogorov–Loveland randomness and stochasticity, Annals of Pure and Applied Logic, vol. 138 (2006), no. 1-3, pp. 183–210

Speaker: Patrick Lutz

Title: Seetapun's theorem and Kolmogorov complexity

Abstract: A theorem implicit in the work of Seetapun and first proved explicitly by Dzhafarov and Jockusch states that for every set A of natural numbers and every noncomputable set X, there is an infinite subset of either A or the complement of A which does not compute X. In essence, this shows that it is impossible to encode an infinite amount of information into all infinite subsets of both a set and its complement. This raises the question: is it possible to encode a finite amount of information? If so, how much? We will give a partial answer to this question within the framework of Kolmogorov complexity and discuss other questions suggested by our results. This is joint work with Matthew Harrison-Trainor.

Speaker: Joe Miller

Title: The Hausdorff dimension of continuous images

Abstract: What effect do continuous functions have on Hausdorff dimension? An uncountable analytic set $E \subset \boldsymbol{R}^2$ must have a perfect subset, so it can be mapped continuously onto $[0,1]^2$---a set of dimension two---regardless of the Hausdorff dimension of E. On the other hand, assuming CH, Patrick Lutz and I constructed a set $E \subset \boldsymbol{R}^2$ of Hausdorff dimension one such that any continuous image of E in $\boldsymbol{R}^2$ has dimension at most one. Moreover, we can ensure that if $f: \boldsymbol{R}^2 \to \boldsymbol{R}^2$ is continuous, then $f[E]$ has Hausdorff dimension zero. The first fact generalizes to any Hausdorff dimension $s \in [0,2]$, but the second does not. Don Stull and I constructed a continuous function $j: R^2 \to \boldsymbol{R}^2$ such that if $E \subset \boldsymbol{R}^2$ has Hausdorff dimension $s > 1$, then $j[E]$ has dimension at least $\frac{s}{2}$. In other words, j (at worst) preserves the relative dimension of sets of Hausdorff dimension greater than one. The work with Partick Lutz implies that, in general, this is best possible.

These results are proved in the effective setting and then ``classicalized'' using the point-to-set principle of Jack Lutz and Niel Lutz.

Speaker: Russell Miller

Title: Computability and Lusin's Theorem

Abstract: Lusin's Theorem states that for every Borel function *f* from reals to reals and every ε>0, there is a continuous function on the reals that is equal to *f* except on a set of measure <ε. A related theorem states that *f* must restrict to a continuous function on some comeager subset of the real line. The analogous statements for functions on the Cantor space are also known to hold.

We give computability-theoretic proofs of these theorems, using the facts that almost all elements *X* of the Cantor space are *generalized low*$_1$, satisfying $X' \leq_T 0' \oplus X$. Here ``almost all'' means both ``all but a set of zero measure'' and ``all but a meager set.'' However, the proofs of these two facts (one for measure, and one for Baire category) are substantially different. We will show how the difference between them accounts for the differences between Lusin's Theorem for measure and the related version for Baire category.

Speaker: Daniel Mourad

Title: Computing Non-Repetitive Sequences Using the Lovász Local Lemma

Abstract: We discuss effective versions of classical results on the existence of non-repetitive sequences first proven using the Lovász Local Lemma, a non-constructive existence result from the probabilistic method. We outline the path to these constructions. First, a probabilistic resample algorithm converges to a witness to the Local Lemma in polynomial expected time. Then, the bound on the expectation is used to build a deterministic algorithm with computable convergence time. However, the resulting effective computation has constraints that make it unsuitable for constructing non-repetitive sequences. We modify the resample algorithm and show that these modifications allow us to relax these constraints.

Speaker: Aristotelis Panagiotopoulos

Title: Universality vs Genericity and C_4-free graphs

Abstract: Let *K* be class of finite structures and let lim**(***K*) be the Polish space of all countable structure on natural numbers which can be ``approximated" by members of *K*. A structure *M* in lim(*K*) is universal if every structure from lim(*K*) embeds in *M* and generic if its isomorphism class is comeager in lim(*K*). The Fraissé construction provides sufficient conditions for lim(*K*) to admit a single member *M* which is both universal and generic. However, there are more general classes *K* for which lim(*K*) contains nonisomorphic structures U and G so that U is universal but not generic and G is generic but not universal. In this talk we will show that the existence of a universal structure implies the existence of a generic structure. We also show that the converse is not true. As a consequence, we provide several new examples of weak Fraïssé classes of finite graphs. Finally, we will show that the class of all countable C_4-free graphs does not contain a generic structure, strengthening a result of A. Hajnal and J. Pach.

This is joint work with Katrin Tent.

Speaker: Christopher Porter

Title: Length Functions And The Dimension Of Points In Self-Similar Fractal Trees

Abstract: In this talk, I will discuss recent work on the effective dimension of points in infinite fractal trees generated recursively by a finite tree over some fixed alphabet. Using unequal costs coding, we can associate a length function with each such fractal tree and show that the channel capacity of the length function is equal to the similarity dimension of the fractal tree (up to a multiplicative constant determined by the size of the alphabet over which our tree is defined). Based on this result, one can further derive formulas for calculating the effective dimension and strong effective dimension of points in fractal trees, establishing analogues of several results due to Lutz and Mayordomo, who studied the effective dimension of points in self-similar fractals in Euclidean space. Finally, I will discuss follow-up work, joint with Adam Case, in which we apply the analogues of the Lutz/Mayordomo results to calculate the change of dimension of sequences under transformations induced by morphism (at least for a sufficiently broad class of sequence).

Speaker: Gihanee Senadheera

Title: Embedding of 1-degrees to PAC/PACi degrees

Abstract: The Probably Approximately Correct (PAC) learning is a machine learning model introduced by Leslie Valiant in 1984. The PAC learnability has a natural reducibility to it. The PACi reducibility refers to the PAC reducibility independent of size and computation time. The reducibility in PAC learning resembles Turing reducibility thus we can define PAC/PACi degrees. If there exists an embedding from 1-degrees to PAC/PACi degrees, then we can infer all the properties of 1-degrees to PAC/PACi degrees. In addition, we can extend this idea to other well-known degrees.

Speaker: Isabella Scott

Title: Effective Constructions of Existentially Closed Groups

Abstract: Existentially closed groups are at the intersection of model theory, computability theory, and algebra. Questions of complexity can be asked in many directions. We will review earlier constructions from the literature and elucidate their computability theoretic power, and propose new constructions with interesting computability theoretic properties.

Speaker: Christopher Shriver

Title: Non-equilibrium Gibbs states on a tree

Abstract: We consider two notions of statistical equilibrium for a probability-measure-preserving shift system: an "equilibrium state" maximizes a functional called the pressure while a "Gibbs state" satisfies a local equilibrium condition. Classical results of Dobrushin, Lanford, and Ruelle show that these notions are equivalent on integer lattices, under some assumptions on the interaction, and the equivalence has been extended to arbitrary amenable groups. Barbieri and Meyerovitch have recently shown that one direction still holds for sofic groups: equilibrium states are always Gibbs.

I will show that the converse fails in a nontrivial way using the example of the free boundary Ising state on the Cayley graph of a free group (an infinite regular tree).

Speaker: Mariya Soskova

Title: Enumeration pointed trees.

Abstract: An enumeration pointed tree (e-pointed tree) is a tree T with no dead ends such that every path in T can enumerate T. This notion arises in work by Montalban [3]: he showed that a degree spectrum of a structure is never the upward closure of an F_σ set in Cantor space or Baire space unless it is the set of Turing oracles that can enumerate an e-pointed tree in Cantor space or Baire space respectively. McCarthy [2] characterized the class of enumeration degrees of e-pointed trees in Cantor space. It turned out to be the well studied class of cototal degrees [1]: degrees of sets A such that A ≤e A. McCarthy also showed that this class is fairly robust: for instance, the degrees of e-pointed trees in Cantor space are the degrees of e-pointed trees with dead ends. We investigate the similar class of e-pointed trees in Baire space and the related class of enumeration degrees of introenumerable sets: a set A is *introenumerable* if it is enumeration reducible to each of its infinite subsets. We prove that the cototal degrees are a strict subclass of the introenumerable degrees, which is in turn strictly contained in the enumeration degrees of e-pointed trees in Baire space. We further characterize the class of e-pointed trees in Baire space with dead ends using the notion of hyperenumeration reducibility, introduced by Sanchis [4]. We show that they strictly extend the degrees of e-pointed trees in Baire space (with no dead ends). This work is joint with Goh, Jacobsen-Groccott, and J. Miller.

[1] Uri Andrews, Hristo Ganchev, Rutger Kuyper, Steffen Lempp, Joseph Miller, Alexandra Soskova, and Mariya Soskova, On cototality and the skip operator in the enumeration degrees, Transactions of the American Mathematical Society, vol. 372(2019), no. 3, pp. 1631–1670.

[2] Ethan McCarthy, Cototal enumeration degrees and their applications to effec- tive mathematics, Proceedings of the American Mathematical Society, vol. 146 (2018), no. 8, pp. 3541–3552.

[3] Antonio Montalban, Computable structure theory—within the arithmetic, Perspectives in Logic, Cambridge University Press, Cambridge; Association for Symbolic Logic, Ithaca, NY, 2021.

[4] Luis Sanchis, Hyperenumeration reducibility, Notre Dame Journal of Formal Logic, vol. 19 (1978), no. 3, pp. 405–415.

Speaker: Hunter Novak Spink

Title: Additive combinatorics at infinity.

Abstract: (Joint with Spencer Dembner) If *X* is an algebraic or o-minimally definable set in C^n, we give a nice description of a set *Y* which together with *exp(X)* unions to the closure of *exp(X)*, answering a question of Gallinaro. The key is a careful extension of the work of Peterzil-Starchenko using infinitessimal stabilizers to answer an analogous question about cocomplete lattice quotient maps of C^n.

Speaker: Greg Terlov

Title: Nonamenable subforests of multi-ended quasi-pmp graphs

Abstract: In the last thirty years, amenability of probability measure preserving (pmp) Borel actions of countable groups has been well understood, largely due to the theory of cost available for pmp countable Borel equivalence relations. On the other hand, very little is known in the quasi-pmp (measure-class preserving) setting, where cost does not yield desirable results. Moreover, since nonamenable groups, such as F_2, can have free amenable quasi-pmp actions, the behavior in this setting has been regarded as particularly mysterious. In this talk, I will present a construction of a nonamenable subforest of multi-ended quasi-pmp Borel graphs. This, together with a result of Tserunyan and Tucker-Drob, witnesses nonamenability of quasi-pmp actions, whose orbit equivalence relations admit such graphings. The main technique is a weighted cycle-cutting algorithm, which yields a weight-maximal spanning forest. We also introduce a random version of this forest, which generalizes the Free Minimal Spanning Forest, to capture nonunimodularity in the context of percolation theory. This is joint work with Ruiyuan Chen and Anush Tserunyan.

Speaker: Manlio Valenti

Title: Exploring some structural properties of the Weihrauch degrees

Abstract: In this talk, we will explore some of the structural properties of the Weihrauch degrees. Despite recent efforts, there are still numerous unanswered questions about the algebraic structure of the Weihrauch lattice. After a brief overview of some known results, I will present some recent discoveries regarding the presence of chains and antichains in the Weihrauch degrees. We will also show how, despite the close interplay between Medvedev and Weihrauch reducibility, the two lattices have a very different structure.

Speaker: Allison Wang

Title: Every CBER is smooth below the Carlson-Simpson generic partition

Abstract: The lowest non-trivial complexity class in the theory of countable Borel equivalence relations (CBERs) is the class of hyperfinite CBERs. One difficulty that arises in studying this class is determining which CBERs are hyperfinite: for many spaces, there are results showing that any CBER is hyperfinite on a "large" set. For instance, a result of Hjorth and Kechris states that every CBER on a Polish space is hyperfinite when restricted to some comeager set. Another result, due to Mathias, shows that every CBER on the Ellentuck Ramsey space is hyperfinite when restricted to some pure Ellentuck cube. In this talk, we will show that every CBER on the space of all infinite partitions of the natural numbers coincides with equality below a Carlson-Simpson generic element. This is joint work with Aristotelis Panagiotopoulos.

Speaker: Amanda Wilkens

Title: Isomorphisms of Poisson systems over locally compact groups

Abstract: We will introduce the Poisson system, a measure-preserving dynamical system made up of a Poisson point process and a group action. Ornstein and Weiss proved Poisson systems over many amenable groups were isomorphic in their 1987 paper. We will define and consider Poisson systems over non-discrete, non-compact, locally compact Polish groups, and prove by construction all Poisson systems over such a group are isomorphic. Additionally, the constructed isomorphism map is finitary, meaning the output Poisson process restricted to a compact set only depends on the input Poisson process restricted to a compact set.