Exam-3 Practice

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1. Determine the behavior of $f(t) = 7(0.13)^t - 8$ as $t \to -\infty$ and $t \to \infty$.
   Answer: As $t \to -\infty$, $f(t) \to \infty$ and as $t \to \infty$, $f(t) \to -8$.

2. Which of the following has largest effective growth rate if the interest is compunded:
   - Annually?
   - Quarterly?
   - Monthly?
   - Continuously?

   Note that the initial value and the rate is same.
   Answer: If the interest is compounded continuously.

3. Convert $240°$ to radians.
   Answer: $\frac{2\pi}{3}$

4. Find the range of $f(t) = 7(7^t + 1)$.
   Answer: $(7, \infty)$

5. Solve for x: $(729)^{x-5} = (1/9)^{2x+5}$.
   Answer: $x = 2$

6. A bee population was introduced in a laboratory where the number of bees is given by:
   $$B(t) = \frac{160}{1 + 5e^{-0.2t}}$$
   Find the time after which the number of bees will be 80.
   Answer: $t = \frac{\ln(0.2)}{-0.2}$.

7. The decomposing rate of an element is 6% per month. Determine its half-life.
   Answer: $\frac{\ln(1/2)}{-\ln(0.06)}$ months

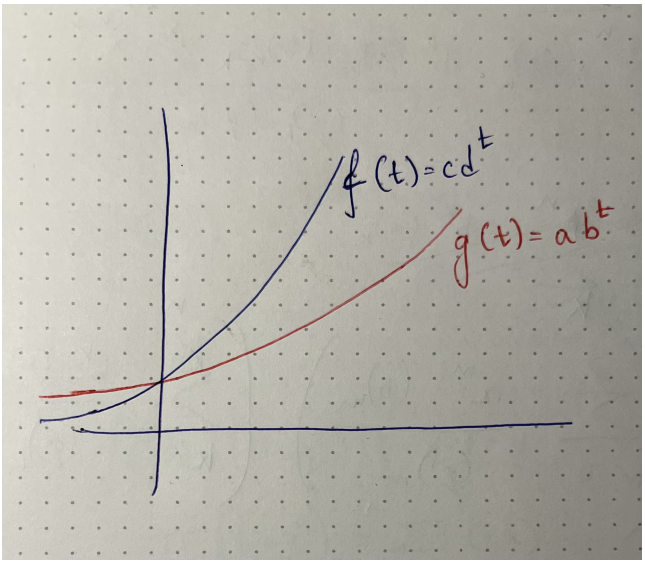
8. Solve for x: $5^{2x-7} + 14 = 20$.
   Answer: $\frac{\log_5(6)+7}{2}$

9. Graph: $f(x) = \log_{22}(x - 22) + 22$

10. Find the values of $\sin(5\pi/6)$ and $\cos(5\pi/6)$. **Answer:** $\frac{1}{2}$, $-\frac{\sqrt{3}}{2}$.

11. Determine TRUE/FALSE:
    $(\log(b))^a = a\log(b)$.
    Answer: FALSE

12. What can you tell about the values b, d from the graph below?



Answer: $b < d$

13. Find a coterminal angle of $-\frac{7\pi}{5}$. **Answer:** $-\frac{17\pi}{5}$ (This is one of the many)

14. Find the effective rate of a continuously compounded growth.
Answer: $e^r - 1$

15. If the population of a country increases by 33.5% per month and reaches 50,000 in 15 years, find the approximate initial population of the country.
Answer: 656

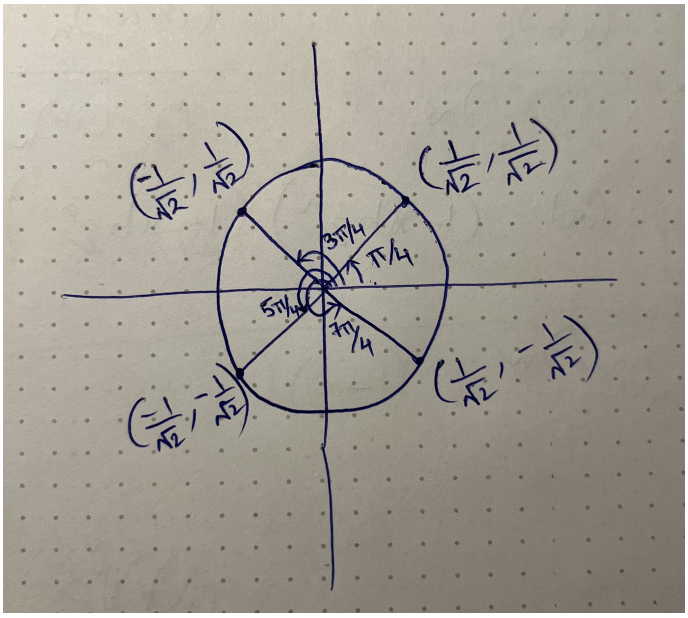
16. Suppose θ is an angle that is symmetric to $\theta - \frac{\pi}{6}$ with respect to $x - axis$.
Answer: $\frac{\pi}{12}$

17. A wheel of a car, with radius 18 units, are rotating 1280° per second. What is the distance travelled by the wheel in one second?
Answer: 128π.

18. For which angle x between 0° and 360°, is $\sin(x) = \sin(45°)$?
Answer: $x = 135°$

19. Solve for x: $\log_3(x) = 2 - \log_3(2x - 3)$.
Answer: $x = 3$.

20. Mark points on the unit circle that make the angles $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ and $\frac{7\pi}{4}$ at the origin. Write down their coordinates. Also, if the $x-$coordinate of a point on the unit circle is $\frac{1}{3}$, then write down its $y-$coordinate.
Answer: Point on unit circle: $(1/3, 2\sqrt{2}/3)$. Solution to the first part is below (next page!).


$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
$\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
$\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
$\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
$3\pi/4$
$\pi/4$
$5\pi/4$
$7\pi/4$