Final Practice

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1. Write down the coordinates of the all points that mark either the maximum or the minimum of the function $f(x) = -3\cos\left(\frac{1}{4}x\right) + 8$, such that the $x-$coordinate lies between -5π and 5π.
   Answer: Points of maxima: $(4\pi, 13), (-4\pi, 13)$, and Points of minima: $(0, 5)$.

2. Let $g(x) = 2x^3 - 5$ and f be given by the following table:

x	$f(x)$
0	2
1	3
-2	-1
3	0
-4	1

   Find $g \circ f$.
   Answer:

x	$g \circ f(x)$
0	11
1	49
-2	-7
3	-5
-4	-3

3. Find the absolute value of the difference between the solutions of the equation $\log_2(x + 1) + \log_2(x - 6) = 3$.
   Answer: 9

4. Find the doubling time of a compound that grows with a continuous growth rate of 13% per year.
   Answer: $\frac{\ln 2}{0.13}$.

5. Find the equation of the line that passes through (3,4) and is perpendicular to the line passing through (-1,1) and (2,-4).
   **Answer:** $3x - 5y + 11 = 0$.

6. Find the interval in which the function $p(x) = x^3 - 2x^2 - x + 2$ is both increasing and concave down. The point of local maxima is $x = \frac{4-2\sqrt{7}}{6}$ and the point of local minima is $x = \frac{4+2\sqrt{7}}{6}$.
   Answer: $\left(-\infty, \frac{4-2\sqrt{7}}{6}\right)$.

7. For which value of t does the equation $\sec^2(x) = t$ has only one solution in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. What is the solution for that particular t?
   Answer: $t = 1, x = 0$.

8. The function L is denoted in the graph below. Find $f^{-1}(4)$.
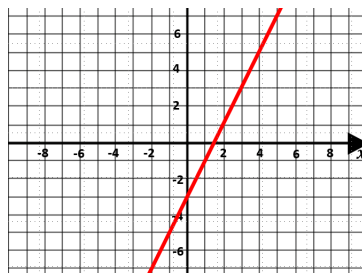
Answer: 5

9. Find the horizontal asymptote of the function: $I(x) = \frac{2x^3 + 10x^2 - 7x}{5x^3 - 9x^2 + x - 11}$.
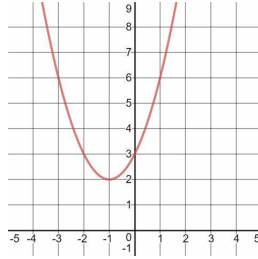   **Answer:** $2/5$.

10. If $xy = 10$ and $x - y = 3$, find all the possible values of $x + y$.
    Answer: ± 7.

11. In which quadrants do the solutions of the equation $(x - 5)^2 + (y + 2)^2 = 9$ lie?
    Answer: I, IV.

12. How many zeros does the following function have? If any, find them.

Answer: 2 zeros; $1 \pm \sqrt{2}i$.

13. An angle of measure $300°$ at the centre of a circle makes an arc of length 10π units. Find the diameter of the circle.
    Answer: 12 units.

14. Find the vertical asymptotes of $2\cot\left(x + \frac{\pi}{3}\right) - 5$.
    Answer: $-\frac{\pi}{3} + \pi k$, where k is an integer.

15. Find the value of $\cos(75°)$.
    Answer: $\sqrt{\frac{2 - \sqrt{3}}{4}}$.

16. Find the value of $\sin(2\operatorname{arccsc}(5/3))$.
    Answer: $\frac{24}{25}$.

17. Find the interval where the following is true:

$$\frac{x^2 - 25}{3 - x} \le 0$$

Answer: $[-5, 3) \cup [5, \infty)$.