

Question List: MAC1147

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1 Class: 08/26

1. (a) Write the following fraction in its simpler form:

$$\frac{125x^7y^3}{5x^85y^2}$$

- (b) Evaluate the simpler form for $x = 20, y = 8$.

2. Evaluate and compare the following:

$$a. \sqrt[3]{-64} \quad b. |\sqrt[3]{-64}| \quad c. \sqrt[6]{-64}$$

3. (a) Find the number of square roots and fourth roots of 625 and list them.

- (b) Does $\sqrt[4]{625}$ give us all the fourth roots of 625?

4. Simplify, for $x = 7, y = 3$,

$$\sqrt[3]{147xy^2}$$

5. Find the perimeter and area of the square each side is $2\sqrt{3}$ long.

6. * Rationalize and compute

$$\sqrt{\frac{48}{5}} + \sqrt{\frac{125}{3}} - \sqrt{\frac{49}{15}}$$

2 Class: 08/29

1. Which of them is a valid polynomial?

$$a. 6 \quad b. X^{\frac{7}{4}} + X^3 + X^{\frac{1}{2}} - \sqrt{10} \quad c. \sqrt{3}X + 3X^2 + 22X^{22}$$

List the degrees, leading coefficients and the constant terms of each of the valid ones.

2. Simplify $(2a + 3b)^3$.

3. Factor out the following polynomial:

$$4b + a^2b^3 - b^4 - 4a^2$$

4. Solve the quadratic equation $x^2 - 5x - 4 = 20$ using:

- (a) Factor Method

- (b) Completion of Square Method

Check whether both the methods give the same solution.

5. * The sum and product of two real numbers are 5 and -10 respectively. Find the numbers.

3 Class: 08/31

1. The largest side of an isosceles right triangle is 5. Find its area.
2. Find five to seven solutions of the following equation and try to draw the graph.

$$|x| + |y| = 1$$

(Hint: Try putting integer values of x , like $x = 0, \pm 1, \pm 2$ and henceforth, and then calculate y).

3. Plot the points $(1,-5)$ and $(-3,2)$ on the Cartesian Plane. Find:
 - (a) the distance between the points
 - (b) the midpoint of the line segment joining the points.
4. Find the equation of the circle whose diameter has endpoints as $(4,3)$ and $(-1,-1)$.
5. Plot the circle with equation $x^2 + y^2 + 6x + 8y - 24 = 0$. Additionally, find its x, y -intercepts.

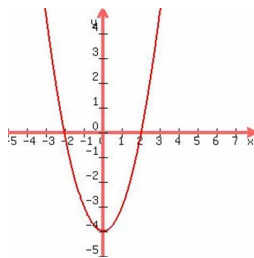
4 Class: 09/02

1. Write down the domain and range of the following relation.

x	y
1	2
2	5
3	10
4	17
5	26
1	2

Is the relation a function? Give reasons to justify your answer. If it is a function, can you give an example of a function on the real numbers that gives the same values as below?

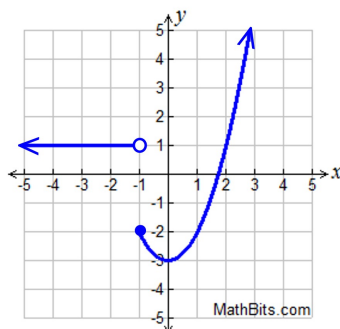
2. Identify the domain and range for the following curve.



3. Find the domain of:

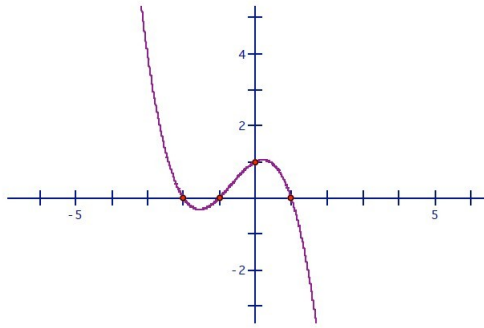
– $f(x) = \frac{5x}{2x-4}$ and $g(x) = \frac{5x}{|2x-4|}$. Is it different for the cases?
– $h(x) = \frac{x-2}{x^2-4}$.

4. Represent the following as a function: x is subtracted from 2 and then the obtained value is divided by 3.
5. Evaluate: $f(-2), f(0), f(2)$

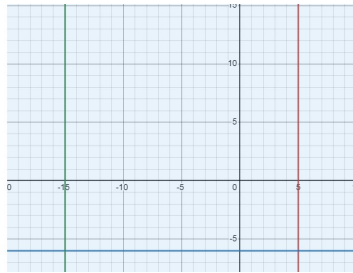


5 Class: 09/07

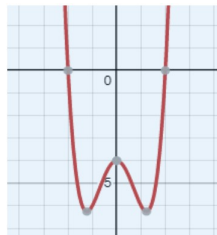
1. Mark the zeros of the function and intervals where the function is positive and negative on its graph.



2. Find the average rate of change of the function $f(x) = x^3 - 8$ on the interval $[2, t]$. Is it a function of t ?
3. Give the intervals where the function is constant, increasing and decreasing.



4. Use the same graph in question 1 and give the points of absolute and relative extremas.
5. Find the point of inflections in the following graph.



6 Class: 09/09

1. $f(x) = 3x + 1$ and $g(x) = 8x^2 + 8x + 1$. Derive the functions $f + g(x)$, $fg(x)$ and $(f/g)(x)$ and find out their respective domains.
2. $f(x) = \frac{1}{x-3}$ and $g(x) = |x - 3|$. Find $gof(x)$ and $fog(x)$. Again, find the domains for both these newly obtained functions.
3. $f(x) = -x$. Find
 - $f \circ f(x)$
 - $f \circ f \circ f(x)$
 - $f \circ f \circ f \circ f(x)$

Now, find a general formula for $f^n(x)$, where $f^n(x) = f \circ f \circ f \circ \dots 'n \text{ times}' \dots of$, i.e. f is composed with itself n times. (Note: The formula will depend on n).

4. * $f(1) = 5, f(2) = 1, f(3) = 3, f(4) = 2, f(5) = 4$. Find the value of n such that $f^n(x) = x$, where f^n has the definition same as that of the previous question.

7 Class: 09/12

1. $f(x) = 3\sqrt[3]{x}$. Shift it towards to right by 3 units and then upwards by 3 units.
2. $f(x) = x^2 + 1$. Vertically reflect and then compress by a factor of 4.
3. $f(x)$ is a function of x . Let $g(x) = 2f(x - 1) + 2$. Comment on the transformation used.
4. Let $f(x) = x^2 + x - 6$. Give the function $g(x)$ obtained by horizontally stretching f by a factor of 3 and then reflecting the new function vertically. Find the zeros of the newly obtained function $g(x)$.
5. Draw the graph of the function $f(x) = -2|x - 1| + 1$.

8 Class: 09/14

1. Is the function $\sin(x)$ invertible?
2. Is the following relation invertible?

x	y
1	2
2	5
3	10
4	17
5	2

Also, find $f^{-1}(10)$ and $(f(4))^{-1}$.

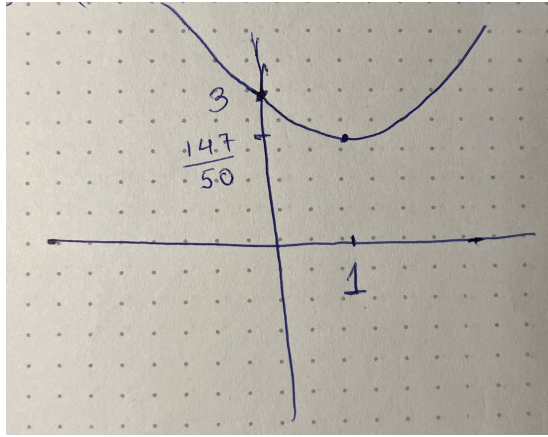
3. Let $f(x) = \frac{x+1}{x-3}$. Find $f^{-1}(0)$.
4. Is $F(x) = \sqrt[2]{-x^2 + 2}$ invertible for every real number? If not, then specify the restricted domain where F is invertible and give the inverse.

9 Lecture 10

1. Find the linear function with x -intercept as 6 and whose graph is parallel to the line passing through (2,4) and (1,-5).
Ans: $f(x) = 9x - 54$
2. How many solutions do the system of equations with lines of same slope have? Give supporting examples for each of your answers.
Ans: Both Zero(Parallel lines) and Infinite(Same line).
3. Is a linear function with non-zero slope always increasing? Give reasons for your answer.
Ans: NO! If slope is negative, then the function can be decreasing.

10 Lecture 11

1. Find the quadratic function with one x -intercept as $1 + 7i$ and a y -intercept 3. Graph the function(use the standard form to graph it).
Ans: $f(x) = \frac{3}{50}(x - 1)^2 + \frac{147}{50}$



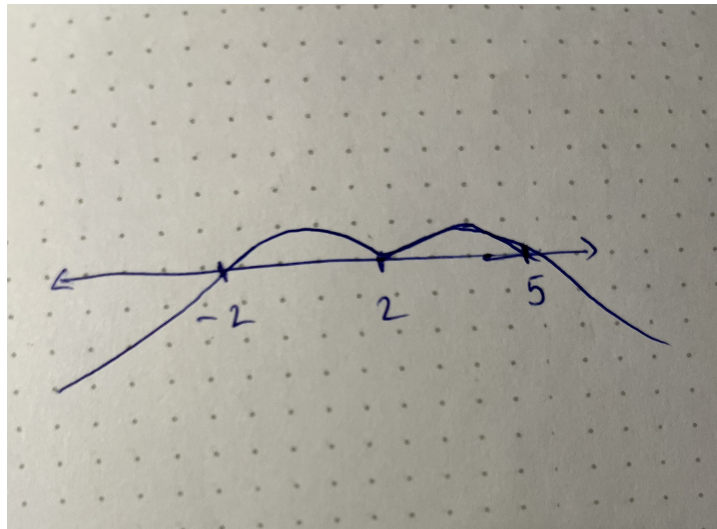
11 Lecture 12

1. What is the maximum number of x -intercepts and turning points possible for the polynomial function $f(n) = -2n^{13} + n^7 + 14n^3 - 23$?

Ans: x -intercepts \rightarrow 13; turning points \rightarrow 12

2. Graph $f(x) = -2(x-2)^2(x+2)^3(x-5)$.

Ans:



3. Find the sum of the rational zeros of $x^3 + x^2 + 5x + 5$.

Ans: -1

12 Lecture 13

1. Combine and simplify $(5 + 7i)(3i - 2) - (-1 - i)$. Identify the real and imaginary parts of the final expression.

Ans: $-30 + 2i$; Real part = -30; Imaginary part = 2

2. Give the condition on complex numbers of the form $a + ib$ for which the complex conjugate of the number is a negative of the number itself.

Ans: $a = 0$

13 Lecture 14-15

1. Find all possible rational zeros of $4x^5 + 3x^3 + 60x + 25$.

Ans: $\pm 1, \pm 5, \pm 25, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{25}{2}, \pm \frac{1}{4}, \pm \frac{5}{4}, \pm \frac{25}{4}$

2. Find the dimension of the cylinder whose height is one less than one half the radius and the volume is 72π cubic meters.

Ans: Radius = 6; Height = 2

3. Solve for x :

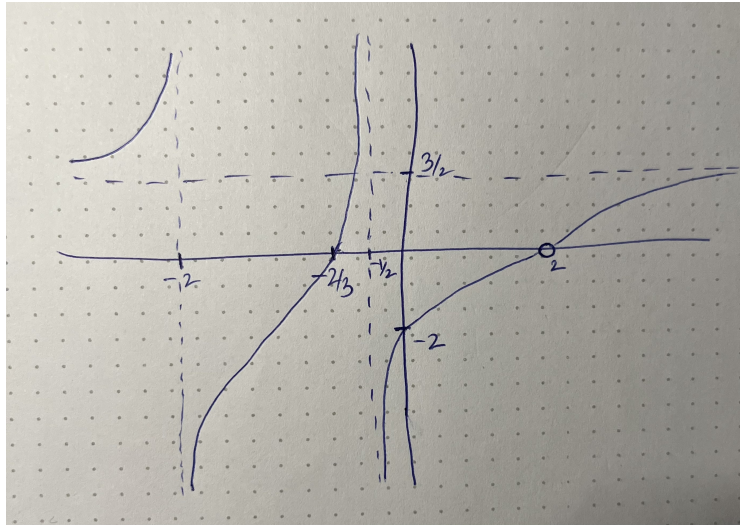
$$\frac{3}{x+2} + \frac{8}{x-3} = 2$$

Ans: $x = \frac{13 \pm \sqrt{17}}{4}$

14 Lecture 16

1. Sketch the graph $f(x) = \frac{3x^3 - 10x^2 + 4x + 8}{2x^3 + x^2 - 8x - 4}$.

Ans: x -intercept = $-\frac{2}{3}$; y -intercept = -2 ; Hole: $(x = 2)$; Vertical Asymptote: $(x = -2, x = -\frac{1}{2})$; Horizontal Asymptote: $(y = \frac{3}{2})$



15 Lecture 17-18

1. Solutions of the inequality $(x + 7)^2(x + 10)(x - 2) < 0$.

Ans: $(-10, 2)$

2. For which values of x does the inequality $|3x + 4| - 8 \leq 3$ hold?

Ans: $[-5, \frac{7}{3}]$

3. If $f(x) = 5x - 3$ and $g(x) = -2x + 8$, then find the values of x for which $f(x) \geq g(x)$.

Ans: $[\frac{11}{7}, \infty)$

4. Find the values of x for which the following inequality holds:

$$\frac{12 - x}{x + 1} \geq 0$$

Ans: $(-1, 12]$

16 Lecture 19

1. Let (a, b) be the solution of the system of equations:

$$\begin{aligned} 2x + y &= 13 \\ x + 5y - 7 &= 0 \end{aligned}$$

Find $a + b$.

Ans: $\frac{59}{9}$

2. Find the solution of the system of equations:

$$\begin{aligned}(x-1)^2 - y + 3 &= 0 \\ y - (x-3)^2 &= 8\end{aligned}$$

Ans: $x = \frac{13}{4}, y = \frac{129}{16}$

3. True or False:

A system of equations consisting of a linear function and a polynomial of three degree can have at least 3 solutions. If your answer is False, write down the correct statement.

Ans: FALSE! Correction: at least will be at most.

17 Lecture 20

1. Let $f(x) = ab^x$ be an exponential function. Give the condition on b such that it is a
- decay function.
 - growth function.

What happens when b does not fall in either of the conditions?

Answer: Decay: $b < 1$; Growth: $b > 1$. If $b = 1$, then $f(x) = a$ is a constant function and hence fails to be an exponential function.

2. A person invested 2,000 dollars in a company shares which made a total profit of 18,000 dollars in 5 years. What was the rate of profit that shares made?
Answer: 14.8 %
3. If the population of a country increases by 33.5% per month and reaches 50,000 in 15 years, find the approximate initial population of the country.
Answer: 656
4. Solve for x : $6^{8x+5} = 36^{3x-7}$.
Answer: $-\frac{19}{2}$

18 Lecture 21

1. An account was open at an initial deposit of 6000 dollars and earns 3.6% interest compounded quarterly. What is the account balance in 15 years?
Answer: NEED TO SOLVE!
2. The formula for the amount A in an investment account with a nominal interest rate r at any time t is given by $A(t) = a(\exp)^{rt}$, where a is the amount of principal initially deposited into an account that compounds continuously. Find the formula for the percentage of interest earned to principal at any time t .
Answer: $f(t) = \exp^{rt} - 1$.
3. A radioactive element decays at a continuous rate of 17.3% per day. Approximately how much amount of the element do we need to obtain 60mg after 3 days.
Answer: 100mg