MAD 6406: Midterm. October 2, 2019

First Name: .... **Last Name: ....**

"On my honor, I have neither given nor received unauthorized aid in doing this assignment."

Signature:..

Directions: Submit solutions to any 3 of the following 4 problems, and clearly indicate on the front page which 3 you would like graded.

No books, no notes, no tablets, no calculators, no computers, no phones!

Write your solutions clearly and legibly for full credit.

Good luck!

#	Points	Score
1	10	
2	10	
3	10	
4	10	
100%	30	

Problem 1. (10 points) Let $A = U\Sigma V^*$ be the singular value decomposition (SVD) of $A \in \mathbb{C}^{m\times n}$ with $\operatorname{rank}(A) = p \le n \le m$.

- **(a)** Show $\{u_1, u_2, \ldots, u_p\}$, is a basis for $\operatorname{Col}(A)$, where $u_1, \ldots, u_p$ are the first p columns of U.
- **(b)** Show $\{u_{p+1}, u_{p+2}, \ldots, u_m\}$ is a basis for $\operatorname{Null}(A^*)$.
- **(c)** Show $\|A\|_2 = \sigma_1$, the first singular value of A.

Problem 2. (10 points) Show the matrix norm equality for $A \in \mathbb{C}^{m\times n}$

$$\|A\|_\infty = \max_{1\le i\le m} \sum_{j=1}^{n} |a_{ij}|$$

Problem 3. (10 points)

- **(a)** Prove that every square matrix A has a Schur decomposition.
- **(b)** Prove that if square matrix A is both normal and upper triangular then it is diagonal.

Problem 4. (10 points) Let $P \in \mathbb{C}^{m\times m}$ be a projector. Show that $\|P\|_2 = 1$ if and only if P is an orthogonal projector.