MAD 6407: Final exam. April 30, 2020
email solutions to s.pollock@ufl.edu by 5p on Thursday, 04/30

First Name: .... **Last Name:** ....

"On my honor, I have neither given nor received unauthorized aid in doing this assignment."

Signature: .... UFID: ....

Directions: Submit solutions to any 4 of the following 5 problems, and clearly indicate on the front page which 4 you would like graded.

You MAY use Matlab for this exam. You MAY NOT consult anyone (other than the course instructor) on any of the problems or solutions.

Write your solutions clearly and legibly for full credit.

Good luck!

#	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
100%	40	

Problem 1. (10 points) Prove for any $f \in C[a,b]$ and integer $n \geq 0$, that the best uniform approximation of f in P_n is unique.

Problem 2. (10 points) Find the values of α and β that minimize each of the following. Interpret your results (this may include an appropriate sketch or two).

$$i. \int_{-1}^{1} (x^4 - (1 + \alpha x + \beta x^2))^2 \, \mathrm{d}\, x$$

$$ii. \int_{-1}^{1} (x^4 - (\alpha x + \beta x^2))^2 \, \mathrm{d}\, x$$

Problem 3. (10 points) Find $x_1, x_2 \in [a,b]$, and $c_1, c_2 \in \mathbb{R}$, that maximize the degree of exactness for the quadrature formula

$$\int_a^b f(x) \, \mathrm{d}\, x \approx c_1 f(x_1) + c_2 f(x_2).$$

Problem 4. (10 points) Consider a finite difference approximation the boundary value problem

$$-u''(x) = f(x), \; 0 < x < 1, \; u'(0) = 0, \; u(1) = 0.$$

Discretize the problem by setting $x_j = jh, \; j = 0, \ldots, n$, where $h = 1/n$. Basing the finite difference formulation on $u''(x) \approx p_2''(x)$ at nodal values, on interior nodes $x_j, \; j = 1, \ldots, n+1$, the approximation $u_j \approx u(x_j)$ should satisfy $h^{-2}(-u_{j-1} + 2u_j - u_{j+1}) = f(x_j)$.

Suppose at the left endpoint x_0 where u should satisfy $u'(x_0) = 0$, the right-looking difference formula based on $p_2'(x_0)$ is used. What is the resulting system of equations? Your answer should be in the form of $AU = F$ where you specify the matrix A, the vector F, and specify the meaning of the vector U.

Problem 5. (10 points) [Computational: Use Matlab, Python (or some other language if you prefer)]

The *Littlewood-Salem-Izumi constant* α_0 is the unique solution $\alpha_0 \in (0,1)$ of

$$\int_0^{3\pi/2} \frac{\cos(t)}{t^\alpha} \, \mathrm{d}\, t = 0.$$

Use Newton's method together with a composite trapezoidal rule with $n = \{10, 100, 1000\}$ subintervals to approximate α_0.