MAD 6407: Midterm. February 28, 2020

First Name: .... **Last Name:** ....

"On my honor, I have neither given nor received unauthorized aid in doing this assignment."

Signature: .... UFID: ....

Directions: Submit solutions to any 4 of the following 5 problems, and clearly indicate on the front page which 4 you would like graded.

No books, no notes, no tablets, no calculators, no computers, no phones!

Write your solutions clearly and legibly for full credit.

Good luck!

#	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
100%	40	

Problem 1. (10 points)

(a) Find the value of α that minimizes

$$\int_0^1 (e^x - (1 + \alpha x))^2 \, \mathrm{d}\, x.$$

(b) Find the values of α and β that minimize

$$\int_0^1 (e^x - (1 + \alpha x + \beta x^2))^2 \, \mathrm{d}\, x.$$

Problem 2. (10 points)

(a) Define the maximum norm $\|f\|_{C[a,b]}$ for a function $f \in C[a,b]$.

(b) Prove for any $f \in C[a,b]$ and integer $n \geq 0$ that the best uniform approximation of f in P_n is unique.

Problem 3. (10 points) The third Chebyshev polynomial T_3 is given by $T_3(x) = 4x^3 - 3x$.

(a) Find the gobal interpolating polynomial on $-2 \leq x \leq 2$ that interpolates $f(x) = e^x$ at the Chebyshev nodes.

(b) Suppose you were given 200 pieces of data, equally spaced on $-2 \leq x \leq 2$, and you want to approximate values that lie between data points. Explain what kind of interpolant you would use to fit these data, and why.

Problem 4. (10 points) Suppose $f \in C^{n+1}[a,b]$, and let $p \in \mathcal{P}_n$ be a polynomial that interpolates the data $\{(x_i, f(x_i)\}_{i=0}^{n}$, where $x_0, \ldots, x_n$, are distict points in $[a,b]$. Consider an arbitrary fixed $x \in [a,b]$, and derive an exact expression for the error $f(x) - p(x)$.

Problem 5. (10 points) Consider the interval $[a,b]$ with the partition $a = x_1 < x_2 < \cdots < x_n < x_{n+1} = b$. Suppose $s(x)$ is the natural cubic spline that interpolates the data $\{(x_i, y_i)\}_{i=1}^{n+1}$, and that $g \in C^2[a,b]$ interpolates the same data. Show that

$$\int_a^b (s''(x))^2 \, \mathrm{d}\, x \leq \int_a^b (g''(x))^2 \, \mathrm{d}\, x.$$