Key

MAC 2312 Quiz 1 September 3, 2024

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. Evaluate $\int_0^{\pi} x^2 \cos(x)\, dx$.

$u = x^2 \quad dv = \cos x\, dx$

$du = 2x\, dx \quad v = \sin x$

$\Rightarrow x^2 \sin x - 2\int x \sin x\, dx \Big]_0^{\pi}$

$u = x \quad dv = \sin x\, dx$

$du = dx \quad v = -\cos x$

$\Rightarrow x^2 \sin x - 2\left[-x\cos x + \int \cos x\, dx\right] \Big]_0^{\pi}$

$= x^2 \sin x + 2x\cos x - 2\int \cos x\, dx \Big]_0^{\pi}$

$= x^2 \sin x + 2x\cos x - 2\sin x \Big]_0^{\pi}$

$= (0 - 2\pi - 0) - (0 + 0 - 0) = \boxed{-2\pi}$

OR

u	dv
x^2 (+)	$\cos x$
$2x$ (−)	$\sin x$
2 (+)	$-\cos x$
0	$-\sin x$

$\Rightarrow x^2 \sin x + 2x\cos x - 2\sin x \Big]_0^{\pi}$

$= \boxed{-2\pi}$

2. Evaluate $\int x \ln x\, dx$.

$u = \ln x \quad dv = x\, dx$

$du = \frac{dx}{x} \quad v = \frac{x^2}{2}$

$\Rightarrow \frac{x^2}{2}\ln x - \frac{1}{2}\int x\, dx$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

OR

$u = x \quad dv = \ln x\, dx$

$du = dx \quad v = x\ln x - x$

$\Rightarrow x^2 \ln x - x^2 - \int (x\ln x - x)\, dx$

$= x^2 \ln x - x^2 - \int x \ln x\, dx + \frac{x^2}{2}$

$\Rightarrow 2\int x\ln x\, dx = x^2 \ln x - \frac{x^2}{2}$

$\Rightarrow \int x \ln x\, dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$