

# Key

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. (4 pts.) Consider the integral

$$\int \frac{1}{\sqrt{x^2 + 4x - 12}} dx.$$

What trigonometric substitution should you use to evaluate this integral? **DO NOT EVALUATE THE INTEGRAL**

Complete the square:  $x^2 + 4x - 12 = x^2 + 4x + 4 - 4 - 12$   
 $= (x+2)^2 - 16$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + 4x - 12}} = \int \frac{dx}{\sqrt{(x+2)^2 - 16}}$$

So the trig sub that should be made is

$$\boxed{x+2 = 4 \sec \theta}$$

2. (6 pts.) Evaluate  $\int \frac{2x+3}{(x+1)^2} dx$ .

$$\frac{2x+3}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\Rightarrow 2x+3 = A(x+1) + B$$

$$2x+3 = Ax + A + B$$

$$x^1: 2 = A$$

$$x^0: 3 = A + B$$

$$= 2 + B$$

$$\Rightarrow B = 1$$

$$\Rightarrow \int \left( \frac{2}{x+1} + \frac{1}{(x+1)^2} \right) dx$$

$$\boxed{= 2 \ln|x+1| - \frac{1}{x+1} + C}$$