Key

MAC 2312 Quiz 3 February 1, 2024

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. (6 *pts.*) Evaluate $\int \frac{3x+4}{(x-3)(x^2+4)}\,dx$.

$$\frac{3x+4}{(x-3)(x^2+4)} = \frac{A}{x-3} + \frac{Bx+C}{(x^2+4)}$$

Cover Up Method: $A = \frac{13}{13} = 1$

$$\Rightarrow \frac{3x+4}{(x-3)(x^2+4)} = \frac{1}{x-3} + \frac{Bx+C}{(x^2+4)}$$

$$\Rightarrow 3x+4 = (x^2+4) + (Bx+C)(x-3)$$
$$= x^2+4+Bx^2-3Bx+Cx-3C$$
$$= (B+1)x^2+(C-3B)x+4-3C$$

x^2: $0 = B+1$, $B=-1$

x^0: $4 = 4-3C$, $C=0$

$$\Rightarrow \int \left(\frac{1}{x-3} - \frac{x}{x^2+4}\right)dx$$

$\frac{1}{x-3}$: direct integration; $\frac{x}{x^2+4}$: u-sub

$$= \ln|x-3| - \frac{1}{2}\ln(x^2+4) + C$$

2. (4 *pts.*) Consider the integral

$$\int \frac{x}{\sqrt{-x^2+10x}}\,dx.$$

What trigonometric substitution should you use to evaluate this integral? **DO NOT EVALUATE THE INTEGRAL**

Complete the square:

$$-x^2+10x = -(x^2-10x)$$
$$= -(x^2-10+25)+25$$
$$= 25-(x-5)^2$$

$\Longleftrightarrow a^2-u^2$

$a = 5$

$u = x-5$

$\Rightarrow x-5 = 5\sin\theta$

$\sin\theta$ substitution