Key

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. Evaluate $\int_{-1}^{\infty} \frac{1}{x^2+6x+13}\,dx$.

Complete the square:

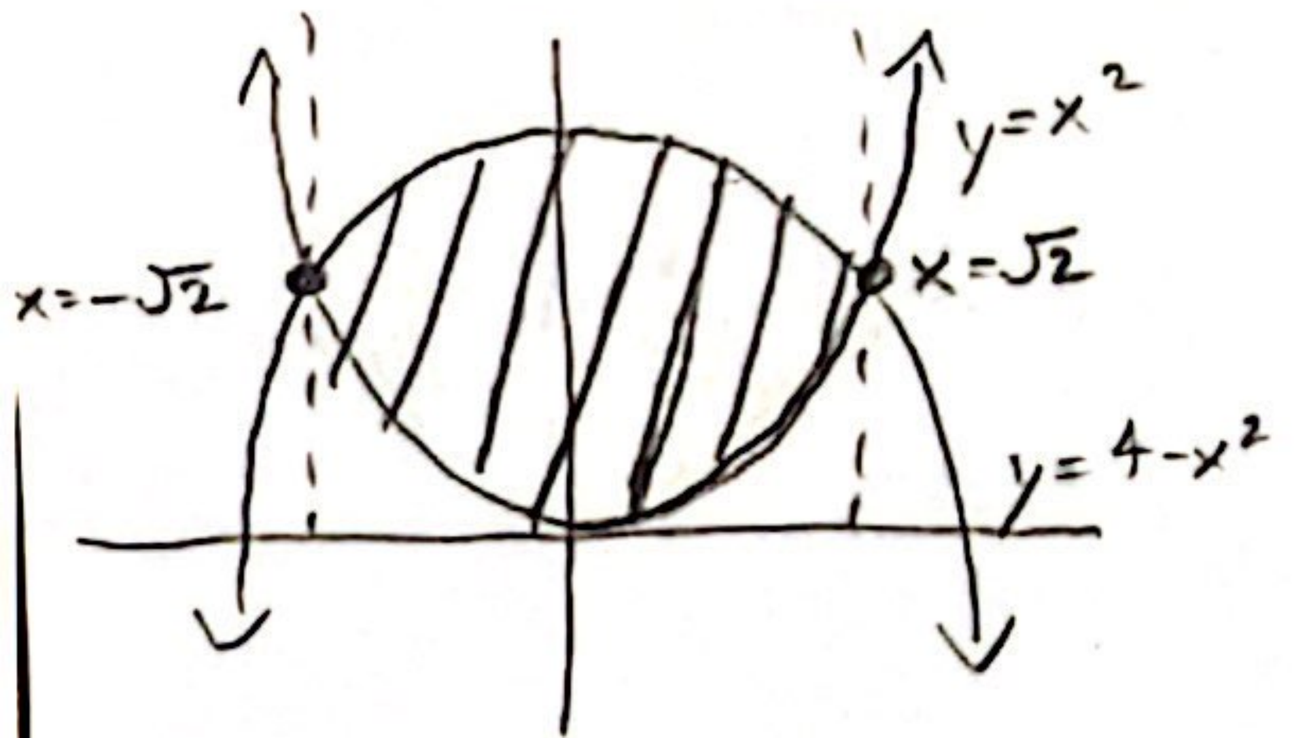
$$x^2+6x+13 = x^2+6x+9+4$$
$$= (x+3)^2+4$$

$$\Rightarrow \lim_{t\to\infty}\int_{-1}^{t}\frac{dx}{(x+3)^2+4} = \lim_{t\to\infty}\frac{1}{2}\arctan\left(\frac{x+3}{2}\right)\Big|_{-1}^{t}$$

trig sub with
$u=x+3$, $a=2$
$\Rightarrow x+3 = 2\tan\theta$
$dx = 2\sec^2\theta\, d\theta$

$$= \lim_{t\to\infty}\left(\frac{1}{2}\arctan\left(\frac{t+3}{2}\right) - \frac{1}{2}\arctan(1)\right)$$
$$= \frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{2}\left(\frac{\pi}{4}\right)$$
$$\boxed{= \frac{\pi}{8}}$$

2. Find the area of the region enclosed between the parabolas $y = x^2$ and $y = 4 - x^2$.



Bounds: $x^2 = 4-x^2$
$2x^2 = 4$
$x^2 = 2$
$x = \pm\sqrt{2}$

Upper: $y = 4-x^2$
Lower: $y = x^2$

$$\Rightarrow \int_{-\sqrt{2}}^{\sqrt{2}} [(4-x^2)-x^2]\,dx$$
$$= \int_{-\sqrt{2}}^{\sqrt{2}} (4-2x^2)\,dx = \left[4x - \frac{2x^3}{3}\right]_{-\sqrt{2}}^{\sqrt{2}}$$
$$= \left(4\sqrt{2} - \frac{4\sqrt{2}}{3}\right) - \left(-4\sqrt{2} + \frac{4\sqrt{2}}{3}\right)$$
$$= 8\sqrt{2} - \frac{8\sqrt{2}}{3}$$
$$\boxed{= \frac{16\sqrt{2}}{3}}$$