

Key

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. Evaluate  $\int_1^2 \frac{\ln x}{x^2} dx$ .

Integration by parts:

$$u = \ln x \quad dv = \frac{1}{x^2} dx$$

$$\Rightarrow du = \frac{1}{x} dx \quad v = -\frac{1}{x}$$

$$\Rightarrow -\frac{\ln x}{x} + \int \frac{1}{x^2} dx \Big|_1^2$$

$$= -\frac{\ln x}{x} - \frac{1}{x} \Big|_1^2 = \left( -\frac{\ln 2}{2} - \frac{1}{2} \right) - \left( -\frac{\ln 1}{1} - 1 \right)$$

$$= \boxed{-\frac{\ln 2}{2} + \frac{1}{2}}$$

2. Evaluate  $\int x^2 e^{-x} dx$ .

Long way:  $u = x^2 \quad dv = e^{-x} dx$

$$\Rightarrow du = 2x dx \quad v = -e^{-x}$$

$$\Rightarrow -x^2 e^{-x} + \int 2x e^{-x} dx$$

IBP again:  $u = 2x \quad dv = e^{-x} dx$

$$\Rightarrow du = 2 dx \quad v = -e^{-x}$$

$$\Rightarrow -x^2 e^{-x} + [-2x e^{-x} + \int 2e^{-x} dx]$$

$$= \boxed{-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C}$$

Tabular method:

u	dv
$x^2$	$e^{-x}$
$2x$	$-e^{-x}$
$2$	$e^{-x}$
$0$	$-e^{-x}$

$$\Rightarrow \boxed{-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C}$$