Key

MAC 2312 Quiz 2 January 30, 2025

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. Evaluate $\int \tan^5 x \sec^3 x \, dx$.

Both powers odd $\Rightarrow$ save a copy of "tanxsecx", set $u = \sec x$, and convert all "$\tan^2 x$" to "$\sec^2 x - 1$"

$$\Rightarrow \int \tan^4 x \sec^2 x \cdot \tan x \sec x \, dx \qquad u = \sec x, \quad du = \tan x \sec x \, dx$$

$$= \int (\tan^2 x)^2 \sec^2 x \cdot \tan x \sec x \, dx$$

$$= \int (\sec^2 x - 1)^2 \sec^2 x \cdot \tan x \sec x \, dx$$

$$= \int (u^2 - 1)^2 u^2 \, du = \int (u^4 - 2u^2 + 1) u^2 \, du$$

$$= \int (u^6 - 2u^4 + u^2) \, du$$

$$= \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C$$

$$= \boxed{\frac{\sec^7 x}{7} - \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + C}$$

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2. Evaluate $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$.

$a^2-u^2 \Rightarrow u = a\sin\theta$

$\Rightarrow x = \sin\theta$

$dx = \cos\theta\,d\theta$

Converting bounds:

$x=0$: $0=\sin\theta$

$\Rightarrow \theta = 0$

$x=1/2$: $1/2=\sin\theta$

$\Rightarrow \theta = \pi/6$

$$\Rightarrow \int_0^{\pi/6} \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\,d\theta$$

$$= \int_0^{\pi/6} \frac{\sin^2\theta}{\sqrt{\cos^2\theta}}\cos\theta\,d\theta$$

$$= \int_0^{\pi/6} \sin^2\theta\,d\theta$$

$$= \int_0^{\pi/6} \frac{1}{2} - \frac{1}{2}\cos 2\theta\,d\theta$$

$$= \frac{\theta}{2} - \frac{1}{4}\sin 2\theta \Big]_0^{\pi/6}$$

$$= \left(\frac{\pi}{12} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right) - (0-0)$$

$$= \boxed{\frac{\pi}{12} - \frac{\sqrt{3}}{8}}$$

Not converting bounds:

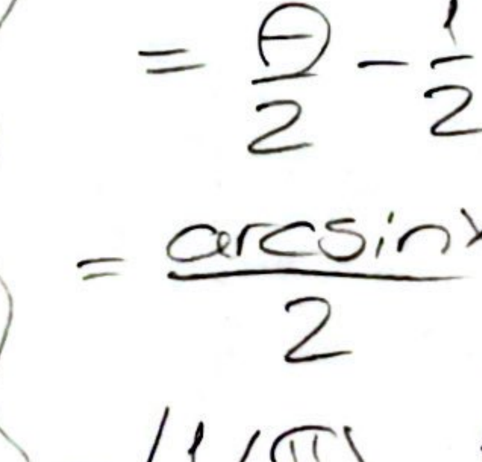
$$\int \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\,d\theta$$

$$= \int \frac{\sin^2\theta}{\sqrt{\cos^2\theta}}\cos\theta\,d\theta$$

$$= \int \sin^2\theta\,d\theta$$

$$= \int \frac{1}{2} - \frac{1}{2}\cos 2\theta\,d\theta$$

$$= \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + C$$



$\Rightarrow \theta = \arcsin x$

Convert θ back to x's:

$$\Rightarrow \frac{\theta}{2} - \frac{1}{4}(2\sin\theta\cos\theta) + C$$

$$= \frac{\theta}{2} - \frac{1}{2}\sin\theta\cos\theta + C$$

$$= \frac{\arcsin x}{2} - \frac{1}{2}x\sqrt{1-x^2}\Big]_0^{1/2}$$

$$= \left(\frac{1}{2}\left(\frac{\pi}{6}\right) - \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\right) - (0-0)$$

$$= \boxed{\frac{\pi}{12} - \frac{\sqrt{3}}{8}}$$