Key

MAC 2312 Quiz 3 February 6, 2025

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. (4 *pts.*) Consider the integral

$$\int \frac{1}{(x^2+4x-5)^{5/2}}\,dx.$$

What trigonometric substitution should you use to evaluate this integral? **DO NOT EVALUATE THE INTEGRAL**

Complete the square:

$x^2+4x-5 = (x^2+4x+4)-5-4$

$= (x+2)^2-9$

$\Rightarrow \int \frac{dx}{((x+2)^2-9)^{5/2}}$

$u^2-a^2 \Rightarrow u = a\sec\theta$

So the substitution is

$\boxed{x+2 = 3\sec\theta}$

2. (6 *pts.*) Evaluate $\int \frac{34}{(x-5)(x^2+9)}\,dx.$

$\frac{34}{(x-5)(x^2+9)} = \frac{A}{x-5} + \frac{Bx+C}{x^2+9}$

Cover-up method to find A:

$A = \frac{34}{5^2+9} = \frac{34}{34} = 1$

Now multiply through equation by $(x-5)(x^2+9)$ to get rid of denominators and substitute in $A=1$:

$34 = (1)(x^2+9)+(Bx+C)(x-5)$

$= x^2+9+Bx^2-5Bx+Cx-5C$

$= (1+B)x^2+(C-5B)x+(9-5C)$

x^2: $0 = 1+B$, $B=-1$

x^0: $34 = 9-5C$, $25 = -5C$, $C=-5$

$\Rightarrow \int \left(\frac{1}{x-5} - \frac{x+5}{x^2+9}\right)dx$

$= \int \left(\frac{1}{x-5} - \frac{x}{x^2+9} - \frac{5}{x^2+9}\right)dx$

① $\int \frac{1}{x-5}\,dx = \ln|x-5|+C$

② $\int \frac{x}{x^2+9}\,dx$, $u = x^2+9$, $\frac{du}{2} = x\,dx$

$\Rightarrow \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln(x^2+9)+C$

③ $\int \frac{5}{x^2+9}\,dx$, $x = 3\tan\theta$

or memorize $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+C$

$\Rightarrow \frac{5}{3}\arctan\left(\frac{x}{3}\right)+C$

$\Rightarrow = \boxed{\ln|x-5| - \frac{1}{2}\ln(x^2+9) - \frac{5}{3}\arctan\left(\frac{x}{3}\right)+C}$