Key

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. Evaluate $\displaystyle\int_0^\infty xe^{-x^2}\,dx$.

$$\Rightarrow \lim_{t\to\infty}\int_0^t xe^{-x^2}dx$$

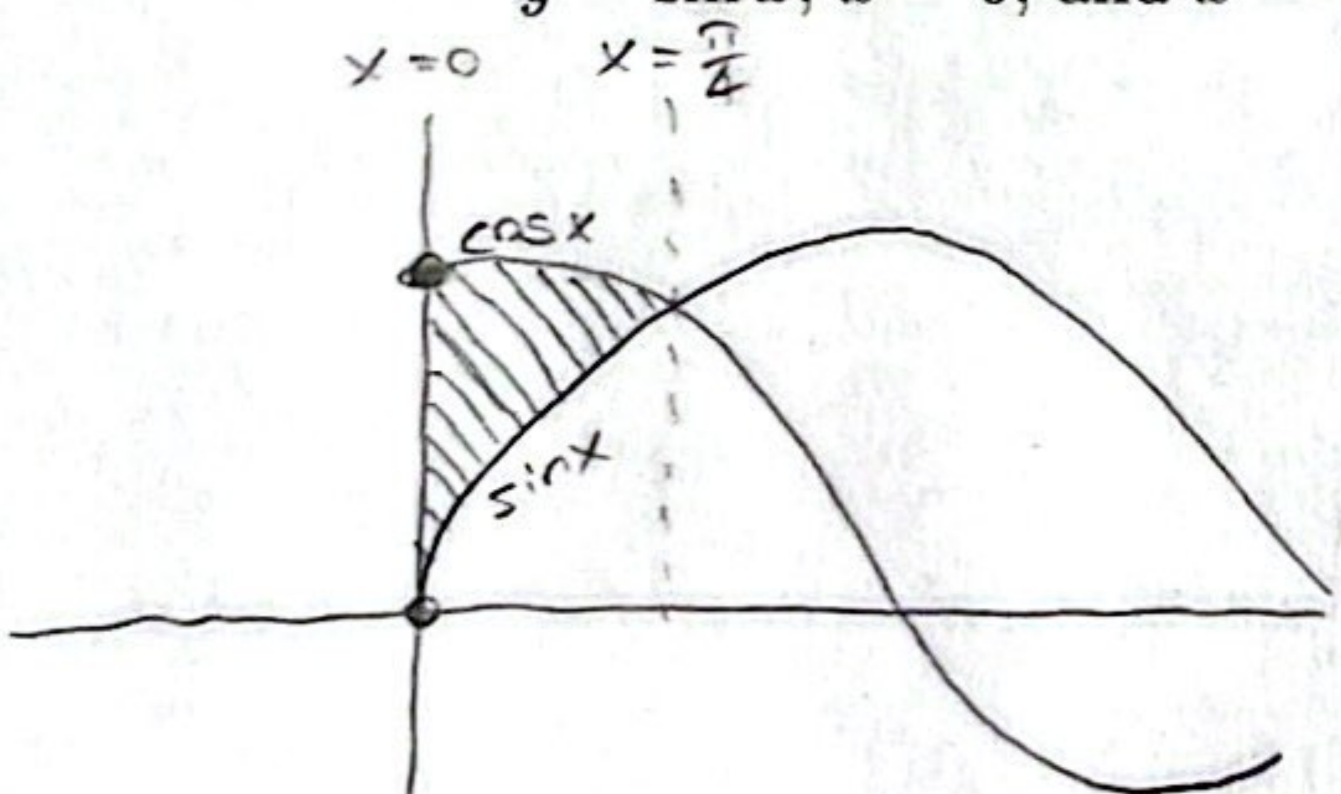
u-sub: $u=-x^2$
$du=-2x\,dx$

$$\Rightarrow \lim_{t\to\infty}\int_0^{-t^2} -\frac{1}{2}e^u\,du$$

$$=\lim_{t\to\infty}\left[-\frac{1}{2}e^u\right]_0^{-t^2} = \lim_{t\to\infty}\left[\cancel{-\frac{1}{2}e^{-t^2}}^{\to 0}+\frac{1}{2}e^0\right]$$

$$=\frac{1}{2}e^0 \boxed{=\frac{1}{2}}$$

2. Find the volume V of the solid generated by revolving the region between the curves $y=\cos x$, $y=\sin x$, $x=0$, and $x=\dfrac{\pi}{4}$ about the x-axis.



Gap between area and axis of revolution $\Rightarrow$ Washer method

$$V=\pi\int_0^{\pi/4}[\cos^2 x-\sin^2 x]dx$$

$$=\pi\int_0^{\pi/4}\left[\frac{1}{2}+\frac{1}{2}\cos(2x)-\left(\frac{1}{2}-\frac{1}{2}\cos(2x)\right)\right]dx$$

$$=\pi\int_0^{\pi/4}\cos(2x)\,dx = \frac{\pi}{2}\sin(2x)\Big|_0^{\pi/4}$$

$$=\frac{\pi}{2}\left(\sin\left(\frac{\pi}{2}\right)-\sin(0)\right)$$

$$=\frac{\pi}{2}(1-0)$$

$$\boxed{=\frac{\pi}{2}}$$