Key

Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. A spring has a natural length of 0.5 meters. If 20 Joules of work is needed to compress the spring 0.1 meters from its natural length, then how much work is required to stretch it from its natural length to a length of 0.7 meters?

$$F_{spring} = kx$$

$$\Rightarrow W_{spring} = \int_a^b kx\, dx$$

To find k:

$$20 = \int_0^{0.1} kx\, dx$$

$$= \frac{k}{2}x^2 \Big|_0^{0.1} = \frac{k}{2}\left(\frac{1}{100}\right)$$

$$\Rightarrow k = 4000$$

Now plug in k to work formula:

$$W = \int_0^{0.2} 4000x\, dx$$

$$= 2000 x^2 \Big|_0^{0.2}$$

$$= 2000\left(\frac{4}{100}\right)$$

$$\boxed{= 80 \text{ J.}}$$

2. Find a formula for the nth term of the sequence below (starting with $n = 1$), and then determine whether the sequence converges or diverges. If the sequence converges, then determine what it converges to.

$$\left\{ 3, -\frac{9}{4}, \frac{27}{16}, -\frac{81}{64}, \dots \right\}$$

$$a_n = \frac{(-1)^{n-1} 3^n}{4^{n-1}}$$

(if you are having trouble finding a_n yourself, contact me.)

$$\Rightarrow \lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{(-1)^{n+1} 3^n}{4^{n-1}}$$

$$= \lim_{n \to \infty} \frac{(-1)^{n-1} 3^{n-1} \cdot 3}{4^{n-1}}$$

$$= \lim_{n \to \infty} \left(-\frac{3}{4}\right)^{n-1} \cdot 3 = 0$$

Therefore the sequence converges to 0.