## ANALYTIC EQUIVALENCE RELATIONS WITH ALL BOREL CLASSES ARE BOREL SOMEWHERE

## WILLIAM CHAN

ABSTRACT. The question of interest in this talk is whether certain equivalence relations on Polish spaces can be a  $\Delta_1^1$  equivalence relation when restricted to certain subsets.

As every equivalence relation is  $\Delta_1^1$  when restricted to countable subsets, this triviality can be removed by requiring the subsets to be non-small according to  $\sigma$ -ideals. This question will be considered for the large class of  $\sigma$ -ideals whose forcing of positive  $\Delta_1^1$  subsets is a proper forcing. A positive answer can only be feasible if the equivalence relations bear at least some resemblance to  $\Delta_1^1$  equivalence relations. Hence, Kanovei, Sabok, and Zapletal asked the following precise question: Is every  $\Sigma_1^1$  equivalence relation with all  $\Delta_1^1$  classes a  $\Delta_1^1$  equivalence relation on some  $\Delta_1^1 I^+$  set, whenever I is a  $\sigma$ -ideal whose associated forcing is a proper forcing.

This talk will discuss this question and how to use a measurable cardinal (or sharps of certain sets) to give a positive answer.

Department of Mathematics, California Institute of Technology, Pasadena, CA 91106 $E\text{-}mail\ address:\ \texttt{wcchan@caltech.edu}$