Speaker: Quinn Culver

Title: The interplay of classes of algorithmically random objects.

Abstract: We study algorithmically random closed subsets (RCS's) of Cantor space, algorithmically random continuous functions (RCF's) from Cantor space to Cantor space, and algorithmically random Borel probability measures on Cantor space; especially the interplay between the three. Our main tools are preservation of randomness (PoR) and its converse no randomness ex nihilo (NREN), which say together that given an a.e.-defined computable map between an effective probability space and an effective polish space, a real is (Martin-Löf) random for the pushforward measure if and only if its preimage is random for the domain's measure. PoR and NREN allow us to prove new facts that were previously open and reprove some known results more simply. Example 1: it was known that every RCS contains a random real and that every random real is contained in some RCS; we reprove this using PoR and NREN. Example 2: It was shown previously that the zero set (that is, the preimage of the sequence of all 0's) of a RCF is a RCS. We reprove this using PoR and get the converse, which was previously left open, that ever RCS is the zero set of some RCF, via NREN. We also prove some miscellaneous results about the individual objects. Example 3: the Lebesgue measure of the range of a RCF is strictly between 0 and 1. This work is joint with Chris Porter.