

Speaker: Rutger Kuyper

Title: A connection between Weihrauch reducibility and intuitionistic reverse mathematics

Abstract: In this talk we will present a contribution to the recent interest in the intuitive similarity between reverse mathematics and Weihrauch reducibility. There are currently two main approaches to formalising this connection. The first of these proceeds by formalising Weihrauch reducibility within reverse mathematics, an approach which was pioneered by Dorais et al. and Dzhafarov.

The other approach is to connect Weihrauch reducibility to intuitionistic reverse mathematics, building on the long-standing intuition that there is a tight connection between intuitionistic logic and computability. This field studying this connection goes under the name of realisability. Techniques from realisability have first been brought into the setting of reverse mathematics by Hirst and Mummert, whose work was continued by Dorais, and by Fujiwara, in their work on the reverse mathematical strength of sequential versions of theorems.

We present a precise connection between Weihrauch reducibility and provability in intuitionistic reverse mathematics, building on the work of the authors mentioned in the previous paragraph. Roughly speaking, we show that a Π^1_2 -statement α implies a second Π^1_2 -statement β over L_0 , the intuitionistic version of RCA_0 , plus Markov's principle, if and only if β Weihrauch-reduces to α . Of course, one quickly realises the theorem cannot be true as stated, but we give a precise way to state the theorem using the notion of composition in the Weihrauch degrees, and we also show that the statement just given holds if we restrict our intuitionistic calculus to an affine variant (i.e. a version which excludes some of the contraction rules in its sequent calculus).