Speaker: José Mijares Palacios

Title: Local Ramsey Theorem. An abstract approach.

Abstract: Given a topological Ramsey space (\mathcal{R}, \leq, r) (see [6]), we extend the notion of semiselective coideal to sets $\mathcal{H} \subseteq \mathcal{R}$ and study conditions for \mathcal{H} that will enable us to make the structure $(\mathcal{R}, \mathcal{H}, \leq, r)$ a Ramsey space (not necessarily topological) and also study forcing notions related to \mathcal{H} which will satisfy abstract versions of interesting properties of the corresponding forcing notions in the realm of Ellentuck's space [2, 4]. As applications, we prove that under suitable large cardinal hypotheses every semiselective ultrafilter $\mathcal{U} \subseteq$ \mathcal{R} is generic over $L(\mathbb{R})$ and that given a semiselective coideal $\mathcal{H} \subseteq \mathcal{R}$, every definable subset of \mathcal{R} is \mathcal{H} -Ramsey. This generalizes some results from [5] and the corresponding results for the case when \mathcal{R} is equal to Ellentuck's space (see [3, 1]).

References

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