Speaker: Russell Miller

Title: Finitary Reducibility on Equivalence Relations

Abstract: A reduction from one equivalence relation E to another one F is a function f for which xEy iff $f(x)Ff(y)$. If f is a Borel function, we call it a *Borel reduction*; while, if f is computable, it is a *computable reduction*. (The fields of E and F must be appropriate to these classes of functions.) Computable reducibility has been the subject of recent investigations by a number of researchers, spurred on by the analogy to Borel reducibility, which has been a standard topic for several decades.

We will discuss joint work with Keng Meng Ng, examining a weaker version of computable reducibility. An *n-reduction* from E to F is a function f which, on input $\langle x_1, \ldots, x_n \rangle$, outputs $\langle y_1, \ldots, y_n \rangle$ such that $x_i E x_j$ iff $y_i F y_j$, and a *finitary reduction* does the same uniformly for all n. Computable finitary reducibility turns out to have interesting properties. For example, it is known from work of Ianovski, Nies, and the two of us that there is no Π^0_2 equivalence relation which is complete among all Π^0_2 equivalence relations under computable reducibility. However, under computable finitary reducibility, the Π^0_2 relation iEj iff $W_i = W_j$ is complete in this sense, and this generalizes nicely to Π^0_m for all $m > 2$. Additionally, for every Turing degree $\boldsymbol{d}$, there exist equivalence relations E and F on ω such that E is computably fi?nitarily reducible to F, but there is no $\boldsymbol{d}$-computable reduction from E to F. Finally, we will give a natural Π^0_2 equivalence relation which is Π^0_2-complete for computable 3-ary reducibility, but not for computable 4-ary reducibility. It remains open to what extent similar results hold for finitary Borel reducibility, and, since Borel reductions deal with equivalence relations on 2^ω, one might also inquire into countable Borel reducibility.