4.4 Uniform Continuity

In this section we present perhaps a new idea to the reader. The property of uniform continuity of a function is perhaps as important as continuity and differentiability and needs to be well understood. We also present yet another class of functions, the Lipschitz functions.

Before giving a formal definition of uniform continuity, let us go back and review continuity on a set E one more time. Recall that Definition 4.1.2 said that a function $f : D \to \Re$ is continuous on a set $E \subseteq D \subseteq \Re$ if and only if for any given $\varepsilon > 0$ and for every point $a \in E$, there exists $\delta > 0$ such that for all points $x \in E$ satisfying $|x - a| < \delta$, we have $|f(x) - f(a)| < \varepsilon$. The δ mentioned in the preceding sentence could vary depending on the choice of a and ε. All this is obvious to the reader by now, but let us prove continuity of a function using the above-quoted definition one more time.