To prove that $f(x) = x^2$, $x \in (-\infty, \infty)$, discussed in Example 4.4.1, is not uniformly continuous, a negation of Definition 4.4.3 needs to be written and then used successfully on the function f. Later, knowing more theory will allow us to complete this task more readily.

Remark 4.4.4. The negation of Definition 4.4.3 is carried out in a way similar to the way that the negation of continuity in Remark 4.1.11, part (b), was done. Here, however, two sequences are involved instead of one, since instead of fixed a we have a free t. Thus, to prove that a given function $f : D \to \Re$ is not uniformly continuous on a set $E \subseteq D \subseteq \Re$, we need to find some particular $\varepsilon > 0$ and two sequences $\{x_n\}$ and $\{t_n\}$ in E that will eventually satisfy $|x_n - t_n| \leq \frac{1}{n}$, but $|f(x_n) - f(t_n)| \geq \varepsilon$. Note that the expression $\frac{1}{n}$ here could be changed to any other expression tending to 0 as n becomes arbitrarily large. □