Unbounded intervals should not be associated with functions that are not uniformly continuous or with bounded intervals with uniformly continuous functions. The function $f(x) = \frac{1}{x}$ with $x \in (0, 1]$ is not uniformly continuous even though its domain is bounded, and the function $f(x) = \sin x$ is uniformly continuous on any domain $D \subseteq \Re$. The validity of these statements will come clear soon. Note also that there are unbounded uniformly continuous functions on unbounded intervals. Can you find one?

Intuitively, uniformly continuous functions do not rise or fall "too quickly" over their domains. If they do rise or fall "very quickly," it is for only a very short period of time. Quadratic functions on unbounded intervals are not uniformly continuous because eventually they rise or fall "too quickly." The uniformly continuous function $f(x) = \sqrt[3]{x}$ from Exercise 1(d) increases extremely quickly as it passes through the origin, but then it quickly levels off.

The theory behind uniformly continuous functions is abundant, and almost the remainder of this section is devoted to such. Uniform continuity is more restrictive than continuity. That is, not every continuous function is uniformly continuous, as verified in Example 4.4.5. Every uniformly continuous function, however, must be continuous as well. Why?