Note that $\forall x \geq c,\ g\left(1-\frac{c}{x}\right) = f(x)$.

Now, fix $\varepsilon > 0$.

$\forall x, y \geq c$, if $|x-y| \leq \delta$ then

$$\left|\left(1-\frac{c}{x}\right)-\left(1-\frac{c}{y}\right)\right| = \left|\frac{c}{y}-\frac{c}{x}\right|$$

$$= \frac{c}{xy}|x-y| \leq |x-y| \leq \delta$$

($\frac{c}{xy} \leq 1$)

$$\Rightarrow |f(x)-f(y)|$$

$$= \left|g\left(1-\frac{c}{x}\right)-g\left(1-\frac{c}{y}\right)\right| \leq \varepsilon.$$

(g unif. cont.)

we conclude that f is unif. cont. on $[c, +\infty)$ (same δ as for the function g).