Exercise: $\sqrt{x}$ is unif. cont. on $[0, +\infty)$.

Solution: Denote $f(x) = \sqrt{x}$.

we know that f is cont. on $[0, +\infty)$, and in particular on $[0, 2]$, which is closed and bounded. Hence, by a th. in class, f is unif. cont. on $[0, 2]$.

- Let $\varepsilon > 0$, then $\forall x, a \geq 1$,

$$|f(x) - f(a)| = |\sqrt{x} - \sqrt{a}|$$

$$= \left| \frac{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}{\sqrt{x} + \sqrt{a}} \right|$$

$$= \frac{|x-a|}{\sqrt{x} + \sqrt{a}} \underset{a, x \geq 1}{\leq} |x-a|.$$

Choose $\delta = \varepsilon$. Then, $\forall x, a \geq 1$,

$$|f(x) - f(a)| \leq |x-a| \leq \delta = \varepsilon.$$