$\Longrightarrow$ f is unif. cont. on $[1, +\infty)$.

we proved that f is unif. cont.

on $[0,2]$ and $[1,+\infty)$:

$\forall \varepsilon > 0$, $\exists \delta_1 > 0$ s.t. $\forall x, a \in [0,2]$, $|x-a| \le \delta_1$

$\Longrightarrow |f(x) - f(a)| \le \varepsilon$

and $\exists \delta_2 > 0$ s.t. $\forall x, a \in [1, +\infty)$, $|x-a| \le \delta_2$

$\Longrightarrow |f(x) - f(a)| \le \varepsilon$.

- Choose $\delta = \min(\delta_1, \delta_2, 1)$. Then,

$\forall x, a \geq 0$, if $|x-a| \le \delta$, then

either $x, a \le 2$ or $x, y \ge 1$

(exercise)

This is because $\delta \le 1$

In each case, $|f(x) - f(a)| \le \varepsilon$.